2. (a) Find the Jacobian $\dfrac{\partial(x,y)}{\partial(s,t)}$ of the mapping defined by

$$\begin{aligned} x &= s^2 e^t \\ y &= s\cos(2t) + \arctan(t^2) \end{aligned}$$

$$\frac{\partial(x,y)}{\partial(s,t)} = \begin{vmatrix} \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \end{vmatrix} = \begin{vmatrix} 2se^t & s^2e^t \\ \cos(2t) & \left[-2s\sin(2t) + \frac{2t}{1+t^4}\right] \end{vmatrix}$$

$$= -4s^2e^t\sin(2t) + \frac{4ste^t}{1+t^4} - s^2e^t\cos(2t)$$

$$\boxed{\frac{\partial(x,y)}{\partial(s,t)} = \frac{4ste^t}{1+t^4} - s^2e^t\left(4\sin(2t) + \cos(2t)\right)}$$

(b) If $z = x - y^2$ where x and y are the functions of s and t given above, use the chain rule to find $\frac{\partial z}{\partial s}$ at the point $(s,t) = (1,0)$. (Hint: Your answer will be a number! Also, you may use that $\arctan(0) = 0$.)

CHAIN RULE: $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = (1)(2se^t) - 2y(\cos(2t))$

At $(s,t) = (1,0)$, $y = (1)\cos(0) + \arctan(0) = 1$

$$\left.\frac{\partial z}{\partial s}\right|_{(s,t)=(1,0)} = 2e^0 - 2(1)\cos(0) = 2 - 2 = 0$$

$$\boxed{\left.\frac{\partial z}{\partial s}\right|_{(s,t)=(1,0)} = 0}$$