

Math 8: Induction

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1. Let $f(x) = 3x + 5$ and let $g(x) = 3(x - 2) + 8$. What's wrong with the following proof?

Fix any integer $n \geq 1$. Assume that $f(n) = g(n)$. Then, $g(n + 1) = 3((n + 1) - 2) + 8 = g(n) + 3 = f(n) + 3 = (3(n) + 5) + 3 = 3(n + 1) + 5 = f(n + 1)$. Therefore, by induction, we have proven $f(n) = g(n)$ for all $n \in \mathbb{N}$.

2. Prove that, for every integer $n \geq 1$,

$$\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$$

3. Prove that, for any fixed real numbers a and $r \neq 1$ that

$$a + ar + ar^2 + \dots + ar^{n-1} = a \left(\frac{r^n - 1}{r - 1} \right)$$

is true for all $n \in \mathbb{N}$.

4. Prove that for all $n \geq 1$, $8^n - 3^n$ is divisible by 5.