Math 8: Integers

Spring 2011; Helena McGahagan

1. Let $a, b \in \mathbb{Z}$. Write the definition of the highest common factor of a and b. Then, negate this definition.

2. (a) Let $a, b, d \in \mathbb{Z}$. Consider the equation $a\,s + b\,t = d$. We want to find integer solutions to this equation (i.e., find $s, t \in \mathbb{Z}$). We have a theorem (a consequence of the Euclidean algorithm) that gives a condition on d that guarantees it is possible to find solutions. What is the condition?

(b) Prove: If $a, b \in \mathbb{Z}$, then any common factor of a and b divides $\text{hcf}(a, b)$.

(c) Consider the equation in part (a) again. If we have a solution (s_o, t_o) (so these numbers satisfy the equation $a\,s_o + b\,t_o = d$), we have infinitely many solutions ... can you find all of them?

3. A warehouse has two trucks; one can hold 28 boxes, and one can hold 34. If the warehouse only sends out the trucks full, and returns them empty, can it move 844 boxes from the warehouse to a store? How many different ways can this be done?

4. Gear A turns at two revolutions per minute. Gear A has 32 teeth and is meshed into Gear B which has 120 teeth. How often will both gears be simultaneously back in their starting positions?

5. If $ac|bc$ and $c \neq 0$, prove that $a|b$.

6. Let $a, b, d \in \mathbb{Z}$. Prove that $\text{hcf}(ad, bd) = |d| \text{ hcf}(a, b)$. (You may assume $d \neq 0$ and a, b are not both 0.)