Math 8: Logic

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Consider the following statements:

P : All natural numbers are even.
$Q : \sqrt{5} > 2$
R : There exists an $n \in \mathbb{N}$ such that $n \geq 500$.

Decide whether each statement is true or false. Then, write the negation of each statement.

Label the following statements as either true or false.

P or $\bar{Q}$ _____ Q and $\bar{R}$ _____

$Q \Rightarrow R$ _____ $P \Rightarrow R$ _____

2. If P and Q are two statements, write the truth table for the statements $P \Rightarrow Q$. and $Q \Rightarrow P$.

Using this, write the truth table for $P \Leftrightarrow Q$. Notice that this statement – "P if and only if Q" – it true only when either both P and Q are true or when both P and Q are false. When $P \Leftrightarrow Q$ is true we often say that P and Q are *equivalent* statements.

Prove that the contrapositive of $P \Rightarrow Q$ is equivalent to $P \Rightarrow Q$.

Using only 'negation', 'and,' and 'or' write a statement that is equivalent to $P \Rightarrow Q$. (Check that the truth table for your statement is exactly the same as the truth table for $P \Rightarrow Q$ – in other words, your statement and "$P \Rightarrow Q$" are either both true or both false!) Can you find an equivalent statement that uses only 'negation' and 'and'?

3. Consider the following two true statements:

P : If an object is a triangle, then it is a polygon

Q : If an number is even, then it is divisible by two.

What are the converses of these statements? Are they true?

What are the contrapositives of these statements? Are they true?