Math 8: Quantifiers

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1. Consider the following statements:

$$P : \forall x \in \mathbb{R}, \exists y \in \mathbb{R} \text{ such that } f(x) = y$$
$$Q : \exists y \in \mathbb{R} \text{ such that } \forall x \in \mathbb{R}, f(x) = y$$

(a) Write the negation of each statement.

(b) Whether or not P and Q are true depend on what f is. What do you need to know about about f to conclude P is true? What do you need to know to conclude that Q is true?

2. (a) Define $A \subseteq B$. Negate this statement

(b) Define $A = B$. Negate this statement.

(c) Negate: $\forall n \in \mathbb{Z}$, $n^2 < e$ or $n^2 > \pi$. Which is true – the original statement or its negation? How can you prove it?

(d) Negate: $\forall n \in \mathbb{Z}$, if $n < 0$, then $n + 5 > 0$. Which is true – the original statement or its negation? How can you prove it?

3. Write the following statements (from your homework) in symbols, then write their negations.

(a) If n is an integer such that n^2 is even, then n is even.

(b) The number 8881 has a prime factor that is at most 89.

(c) An integer that is equal to three consecutive integers multiplied together is divisible by 3.