

Math 8: Homework 3

Read Chapter 10.

Exercises: Hand in all of the following in lecture on Thursday, April 21th.

Chapter 10: #1, #2, #4, #6, #8, #10

Chapter 11: #4

I. Use the symbol A for ten and B for eleven. Write $(1AB9)_{12}$ in base 10. Use the division algorithm to convert the integer 1511 to base 12.

II. For which of the following equations can we find $s, t \in \mathbb{Z}$ that satisfy the equation? (Either give a proof that the equation can be solved or prove that it cannot have a solution.)

(a) $s \cdot 11 + t \cdot 17 = 120$

(b) $s \cdot 14 + t \cdot 18 = 120$

(c) $s \cdot 14 + t \cdot 18 = 30$

III. Prove or disprove: $\text{hcf}(a, b) = \text{hcf}(a + b, \text{lcm}(a, b))$.

IV. Let $a \in \mathbb{N}$. Assume b and c are odd prime numbers and $a < b < c$. Prove that if $a \mid (3b + 2c)$ and $a \mid (2b + 3c)$, then either $a = 1$ or $a = 5$. Give an example of odd prime numbers b and c such that $5 \mid (3b + 2c)$ and $5 \mid (2b + 3c)$.