

Math 8: Homework 4

Read Chapter 11 and 13.

Exercises: Hand in all of the following in lecture on Tuesday, May 3rd.

Chapter 11: #6; #8

Chapter 17: #6

Chapter 12: #4

Chapter 13: #2(a); #4(a) and #5(a); #6; #8; #10

- I.** (a) Find an integer $0 \leq r < 3$ such that $8^{24} = r \pmod{3}$.
(b) Find an integer $0 \leq r < 7$ such that $10^{45} = r \pmod{7}$.
(c) Find the last digit of $3^{7^{29}}$.
(d) Is $6^{17} + 17^6$ divisible by either by 3 or by 7?

II. For each of the following congruence equations, either find a solution $x \in \mathbb{Z}$ or prove that there is no solution.

- (a) $4x \equiv 6 \pmod{14}$
(b) $x^2 \equiv (6x - 1) \pmod{8}$
(c) $x^2 \equiv (6x + 3) \pmod{5}$
(d) $x^{11} + x^7 + x^5 + 1 \equiv 0 \pmod{2}$
(e) $x^9 + x^3 + 1 \equiv 0 \pmod{2}$

III. A book's ISBN-10 number is a 10-digit number, separated into four groups (for example, 0-123-45678-9). The first group of digits represents the book's language, the next group represents the publisher, and the third group represents the publisher's number for the book. The final digit is always a *check digit* that can be used to help verify that the number is copied correctly. If the number $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}$ is a valid ISBN-10 number, then the final digit must be the remainder (modulo 11) of $1a_1 + 2a_2 + 3a_3 + \dots + 9a_9$: that is,

$$1a_1 + 2a_2 + 3a_3 + \dots + 9a_9 \equiv a_{10} \pmod{11}.$$

(Since we are using mod 11, the digit a_{10} can be 0, 1, 2, ..., 9, or X , where X represents 10.) Answer the following questions about ISBN-10 numbers (show your work).

- (a) Is 0-763-52302- X a valid ISBN-10 number?
(b) Is 1-33-003151-5 a valid ISBN-10 number?
(c) Find the missing check digit in the following ISBN-10 number: 13-210-0056-?
(d) Find the missing check digit in the following ISBN-10 number: 2-410-11282-?