Math 8: Homework 5

Review Chapter 14 and read Chapter 8.

Exercises: Hand in all of the following in lecture on Thursday, May 12th.

Chapter 14: #2, #4, #6

Chapter 8: #2, #10(a), #12

I.
(a) Find $4^{403} \mod 11$.

(b) Prove that for all $n \in \mathbb{Z}$, $n^{91} - n^7$ is divisible by 91.

II.
(a) Find all integers k such that $2^k \equiv 1 \mod 11$.

(b) Find all integers x such that $x^{11} \equiv 2 \mod 59$.
   (Hint: 11 and 58 are coprime; start by finding *positive* integers s, t that solve $11s - 58t = 1$.)

(c) Use Fermat's little theorem to find the inverse of $[4]$ in $\mathbb{Z}_{13}$.

III. The notation $\displaystyle\prod_{r=1}^{n} a_r$ is shorthand for the product of the numbers: $a_1 \cdot a_2 \cdot a_3 \cdots a_n$.

Guess a simple formula (depending on n) for $\displaystyle\prod_{r=2}^{n} \left(1 - \frac{1}{r^2}\right)$ and prove that your formula is correct.