Math 8: Homework 6

Review Chapter 8 and read Chapter 16, pages 131-133 (on the binomial theorem). Also, start reading Chapter 18 (on equivalence relations)

Exercises: Hand in all of the following in lecture on Thursday, May 19^{th}.

Chapter 8: #14

Chapter 16: #4

Chapter 18: #1

I. (a) Use the binomial theorem to expand $(x + \frac{1}{x})^8$.

(b) Use the binomial theorem to expand $(4x^2 - 3y^3)^4$.

II. Use induction to prove that the size of the power set $|P(A)| = 2^{|A|}$ for all finite sets A. (Hint: Induct on the number of elements in the set A.)

III. Use induction to prove the "Pigeonhole Principle": That is, for all $n \in \mathbb{N}$, prove the statement "If $n+1$ objects are placed in n boxes, then one box must contain at least two objects."

IV. The Fibonacci sequence is the sequence of integers $f_1, f_2, f_3, ...$ defined by $f_1 = 1$, $f_2 = 1$, and for $n \geq 3$, $f_n = f_{n-1} + f_{n-2}$.

(a) Write down the first 10 numbers in the Fibonacci sequence.

(b) Prove that $f_{n+1} < \left(\frac{7}{4}\right)^n$ for all $n \in \mathbb{N}$.

V. (a) Find an example of a relation $\sim$ on a set S that is symmetric and transitive, but not reflexive.

(b) Clearly, the following proof must be wrong since (a) gives a counterexample – what's the flaw?

"Let S be a set and let $\sim$ be a symmetric and transitive relation on S. For any $a, b \in S$, $a \sim b$ implies that $b \sim a$ because $\sim$ is symmetric. But $a \sim b$ and $b \sim a$ imply that $a \sim a$ because $\sim$ is transitive. Since $a \sim a$, the relation $\sim$ must be reflexive."