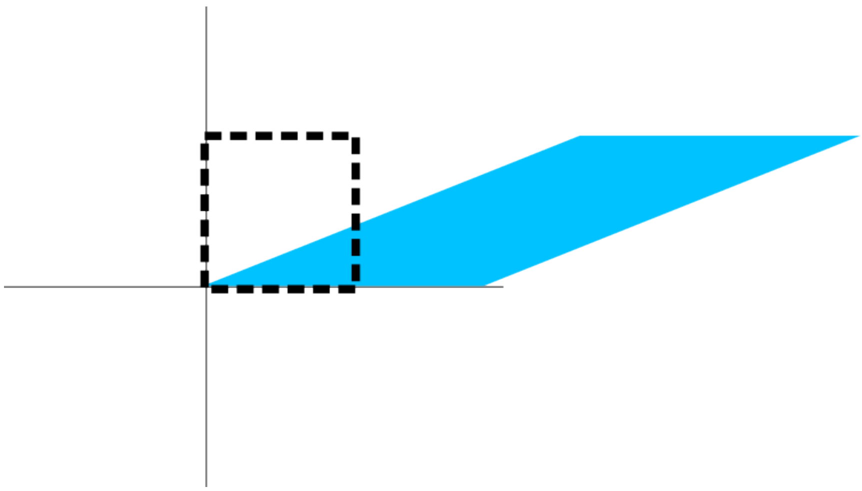
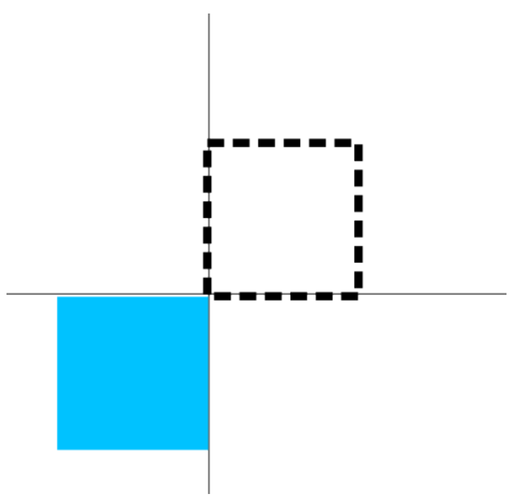
1.9.8 $T : \mathbb{R}^2 \to \mathbb{R}^2$ first performs a horizontal shear that transforms $\mathbf{e}_2$ into $\mathbf{e}_2 + 2\mathbf{e}_1$ (leaving $\mathbf{e}_1$ unchanged) and then reflects points through the line $x_2 = -x_1$. Find the standard matrix of T.

This problem is mostly testing your knowledge of different geometric transformations of $\mathbb{R}^2$. It will be easiest if we treat these one at a time.

Lets call S the shear. $S(\mathbf{e}_2) = \mathbf{e}_2 + 2\mathbf{e}_1$. S is a *horizontal* shear, so it doesn't do anything to the x-axis. So $S(\mathbf{e}_1) = \mathbf{e}_1$.



Lets call R the reflection. Reflecting $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ across the line $x_2 = -x_1$ gives the point $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$. Similarly, reflecting $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ across the line $x_2 = -x_1$ gives the point $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$. So $R(\mathbf{e}_1) = -\mathbf{e}_2$ and $R(\mathbf{e}_2) = -\mathbf{e}_1$.



T is the transformation where we do first S, then R. (*You will usually see this written $T = R \circ S$, read as "R compose S"*). So

$$\begin{aligned} T(\mathbf{e}_1) &= R(S(\mathbf{e}_1)) \\ &= R(\mathbf{e}_1) \\ &= -\mathbf{e}_2. \end{aligned}$$

Likewise,

$$\begin{aligned} T(\mathbf{e}_2) &= R(S(\mathbf{e}_2)) \\ &= R(\mathbf{e}_2 + 2\mathbf{e}_1) \\ &= R(\mathbf{e}_2) + 2R(\mathbf{e}_1) \\ &= -\mathbf{e}_1 - 2\mathbf{e}_2. \end{aligned}$$

At this point we can compute the standard matrix in the usual way. The first column will be the vector $-\mathbf{e}_2$, and the second column will be the vector $-\mathbf{e}_1 - 2\mathbf{e}_2$. Thus the standard matrix is given by

$$\begin{bmatrix} 0 & -1 \\ -1 & -2 \end{bmatrix}.$$