1.9.8 \( T : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) first performs a horizontal shear that transforms \( \mathbf{e}_2 \) into \( \mathbf{e}_2 + 2\mathbf{e}_1 \) (leaving \( \mathbf{e}_1 \) unchanged) and then reflects points through the line \( x_2 = -x_1 \). Find the standard matrix of \( T \).

This problem is mostly testing your knowledge of different geometric transformations of \( \mathbb{R}^2 \). It will be easiest if we treat these one at a time.

Let's call \( S \) the shear. \( S(\mathbf{e}_2) = \mathbf{e}_2 + 2\mathbf{e}_1 \). \( S \) is a horizontal shear, so it doesn't do anything to the \( x \)-axis. So \( S(\mathbf{e}_1) = \mathbf{e}_1 \).

Let's call \( R \) the reflection. Reflecting \( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) across the line \( x_2 = -x_1 \) gives the point \( \begin{bmatrix} 0 \\ -1 \end{bmatrix} \). Similarly, reflecting \( \begin{bmatrix} 0 \\ 1 \end{bmatrix} \) across the line \( x_2 = -x_1 \) gives the point \( \begin{bmatrix} -1 \\ 0 \end{bmatrix} \). So \( R(\mathbf{e}_1) = -\mathbf{e}_2 \) and \( R(\mathbf{e}_2) = -\mathbf{e}_1 \).
$T$ is the transformation where we do first $S$, then $R$. (You will usually see this written $T = R \circ S$, read as “$R$ compose $S$”). So

$$T(e_1) = R(S(e_1))$$
$$= R(e_1)$$
$$= -e_2.$$  

Likewise,

$$T(e_2) = R(S(e_2))$$
$$= R(e_2 + 2e_1)$$
$$= R(e_2) + 2R(e_1)$$
$$= -e_1 - 2e_2.$$  

At this point we can compute the standard matrix in the usual way. The first column will be the vector $-e_2$, and the second column will be the vector $-e_1 - 2e_2$. Thus the standard matrix is given by

$$\begin{bmatrix} 0 & -1 \\ -1 & -2 \end{bmatrix}.$$