

Math 4B:  
Section 2, Jan 11th. Time:

Solve the following IVP. Sketch their solutions and determine approximately the interval of existence for each.

$$y' = (1 - 2x)y^2 \quad (1)$$

$$y(0) = -1/6 \quad (2)$$

$$y' y^2 = 1 - 2x$$

$$\int y^2 dy = \int (1 - 2x) dx$$

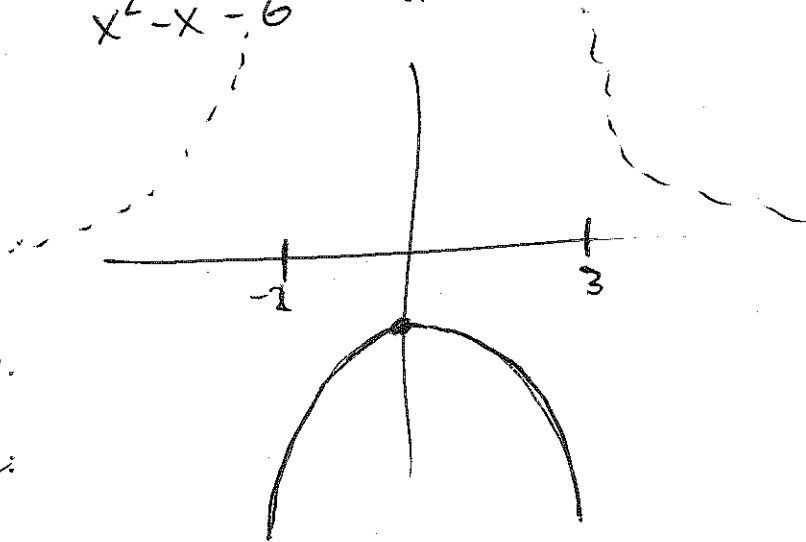
$$-y^{-1} = x - x^2 + C$$

use I.C.:

$$-(-6) = 0 + C \Rightarrow C = 6$$

Interval of existence:  
 $t \in (-2, 3)$

$$y = \frac{1}{x^2 - x - 6} = \frac{1}{(x-3)(x+2)}$$



$$y' = (3x^2 - e^x)/(2y - 5) \quad (3)$$

$$y(0) = 1 \quad (4)$$

$$(2y - 5)y' = 3x^2 - e^x$$

Integrate

$$y^2 - 5y = x^3 - e^x + C$$

use I.C.

$$1 - 5 = 0 - 1 + C$$

$$C = -3$$

Plot with computer

=>

Interval of existence

$$\approx (-1.4, 4.6)$$

Solve the following IVP.

$$y' + 2y = te^{-2t} \quad (5)$$

$$y(1) = 0 \quad (6)$$

$$\mu(t) = e^{2t}$$

$$e^{2t} y' + 2e^{2t} y = t$$

$$[e^{2t} y]' = t$$

$$y = \left(\frac{t^2}{2} - \frac{1}{2}\right)e^{-2t}$$

$$e^{2t} y = \frac{t^2}{2} + c$$

Use i.c.:

$$c = -\frac{1}{2}$$

$$ty' + (t+1)y = t \quad (7)$$

$$y(\ln 2) = 1, \quad t > 0 \quad (8)$$

$$y' + \left(1 + \frac{1}{t}\right)y = 1$$

$$\mu(t) = e^{t + \ln t} = te^t$$

$$te^t y' + (t+1)e^t y = te^t$$

$$y = 1 - \frac{1}{t} + \frac{2}{te^t}$$

$$[te^t y]' = te^t$$

$$te^t y = (t-1)e^t + c$$

use i.c.

$$c = 2$$