Worksheet 1

1.1.9 The following augmented matrix of a linear system has been reduced by row operations to the form shown. Describe the solution set of the original system.

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -5 \\ 0 & 1 & -2 & 0 & -7 \\ 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right].$$

1.1.16 Write the following system as an augmented matrix and reduce it to row echelon form. Is this system consistent?

$$\begin{array}{rrrrcr} 2x_1 & & & -4x_4 & = & -10 \\ & 3x_2 & +3x_3 & & = & 0 \\ & & x_3 & +4x_4 & = & -1 \\ -3x_1 & +2x_2 & +3x_3 & +x_4 & = & 5 \end{array}$$

Suppose we have a linear system. What happens to the solution set if we multiply one of the equations by 0? Find one example where this doesn't change the solution set, and one example where it does. (*This is why, when performing row operations, we only multiply rows (equations) by nonzero scalars. Multiplying a row (equation) by a nonzero scalar always preserves the solution set.*)

1.1.19,1.1.22 Determine the value(s) of h such that

$$\left[\begin{array}{cc|c} 1 & h & 4 \\ 3 & 6 & 8 \end{array}\right]$$

is the augmented matrix of a consistent linear system. Justify your answer.
Do the same with

$$\left[\begin{array}{cc|c} -4 & 12 & h \\ 2 & -6 & -3 \end{array}\right].$$

1.1.25 Find an equation involving g, h, and k which makes this augmented matrix correspond to a consistent system:

$$\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right].$$

Consider the augmented matrix

$$A = \left[\begin{array}{cccc|c} 0 & 2 & -2 & 4 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 4 & 1 \end{array}\right].$$

Does this correspond to a consistent system of equations? If so, does this system have a *unique* solution?

In your book, you have seen that solutions can be written in a parametric form. Consider, for instance, the system

$$\begin{aligned} x_1 \quad -5x_3 &= 1 \\ x_2 \quad +x_3 &= 4 \\ 0 &= 0. \end{aligned} \tag{1}$$

The solution set can be written as

$$\begin{cases} x_1 = 1 + 5x_3 \\ x_2 = 4 - x_3 \\ x_3 \text{ is free.} \end{cases} \tag{2}$$

Lay also remarks "whenever a system is consistent and has free variables, the solution set has many parametric descriptions." (p.19) For instance, write the solution set to (1) where x_1 is the parameter, rather than x_3. Likewise, write the solution set using x_2 as the parameter.

In this case, the parameterization can be given in terms of *any* of the variables. Let us see that this is not always the case. Find a system for which some but not all of the variables can be used to parameterize the solution. (*This shows that the choice of parameters is largely one of convention. Note that in this course, we will always parameterize systems in terms of their free variables.*)