Worksheet 2

Find the solution set of the system corresponding to

$$\begin{bmatrix} 0 & 1 & 2 & | & 5 \\ 0 & -1 & 3 & | & 25 \end{bmatrix}$$

in the form

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix} + s \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Does the augmented matrix

$$\begin{bmatrix} 1 & 2 & -1 & | & 3 \\ 0 & 1 & 1 & | & 2 \\ 1 & 3 & 0 & | & 4 \end{bmatrix}$$

have zero, one, or infinitely many solutions?

1.3.12 Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$:

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

1.3.22 Construct a 3×3 matrix A, with nonzero entries, and a vector $\mathbf{b}$ in $\mathbb{R}^3$ such that $\mathbf{b}$ is *not* in the set spanned by the columns of A.

1.4.15 Let $A = \begin{bmatrix} 3 & -1 \\ -9 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Show that the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for all possible $\mathbf{b}$, and describe the set of $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ does have a solution.

1.4.26 Let $\mathbf{u} = \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$. It can be shown that $2\mathbf{u} - 3\mathbf{v} - \mathbf{w} = \mathbf{0}$. Use this fact (and no row operations) to find x_1 and x_2 that satisfy the equation

$$\begin{bmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$$

1.4.32 Could a set of 3 vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ where n is less than m?

In

$$A = \begin{bmatrix} 4 & 1 & 5 \\ -5 & 3 & -2 \\ -2 & -1 & -3 \\ 1 & 0 & 1 \end{bmatrix}$$

note that one column is the sum of the other two. Find three different solutions to $A\mathbf{x} = \mathbf{0}$. Note that you *do not* need to use row operations for this problem. See 1.4.26 for inspiration.