Worksheet 3

1.4.13 Let $u = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$. Is u in the plane in $\mathbb{R}^3$ spanned by the columns of A?

1.5.6 Provide a vector description of the set of all solutions to

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 0 \\ 2x_1 + x_2 - 3x_3 &= 0 \\ -x_1 + x_2 \quad\quad &= 0 \end{aligned}$$

1.5.33 Construct a 3x3 nonzero matrix A such that the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a solution of $A\mathbf{x} = \mathbf{0}$.

1.7.10 Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ -5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 9 \\ 15 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ -5 \\ h \end{bmatrix}$ For what values of h is $\mathbf{v}_3$ in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$, and for what values of h is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly dependent? Justify each answer.

1.5.36 Given $A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \\ 12 & -8 \end{bmatrix}$, find one nontrivial solution of $A\mathbf{x} = \mathbf{0}$ by inspection.

1.5.28,1.5.30 For each part, determine if $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, and also determine if $A\mathbf{x} = \mathbf{b}$ has at least one solution for every possible $\mathbf{b}$.

- If A is a 3x3 matrix with three pivot positions?
- If A is a 2x5 matrix with two pivot positions?

Can you find numbers m, n, and k such that any $m \times n$ matrix A with k pivot positions such that the system $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions, yet $A\mathbf{x} = \mathbf{b}$ does not necessarily have a solution for every possible $\mathbf{b}$?