Worksheet 3

1.4.13 Let \( u = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix} \) and \( A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix} \). Is \( u \) in the plane in \( \mathbb{R}^3 \) spanned by the columns of \( A \)?

1.5.6 Provide a vector description of the set of all solutions to

\[
\begin{align*}
    x_1 + 2x_2 - 3x_3 &= 0 \\
    2x_1 + x_2 - 3x_3 &= 0 \\
    -x_1 + x_2 &= 0
\end{align*}
\]

1.5.33 Construct a 3x3 nonzero matrix \( A \) such that the vector \( \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \) is a solution of \( Ax = 0 \).

1.7.10 Let \( v_1 = \begin{bmatrix} 1 \\ -3 \\ -5 \end{bmatrix}, v_2 = \begin{bmatrix} -3 \\ 9 \\ 15 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -5 \\ h \end{bmatrix} \). For what values of \( h \) is \( v_3 \) in \( \text{Span}\{v_1, v_2\} \), and for what values of \( h \) is \( \{v_1, v_2, v_3\} \) linearly dependent? Justify each answer.

1.5.36 Given \( A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \\ 12 & -8 \end{bmatrix} \), find one nontrivial solution of \( Ax = 0 \) by inspection.

1.5.28,1.5.30 For each part, determine if \( Ax = 0 \) has a nontrivial solution, and also determine if \( Ax = b \) has at least one solution for every possible \( b \).

- If \( A \) is a 3x3 matrix with three pivot positions?
- If \( A \) is a 2x5 matrix with two pivot positions?

Can you find numbers \( m, n, \) and \( k \) such that any \( m \times n \) matrix \( A \) with \( k \) pivot positions such that the system \( Ax = 0 \) has nontrivial solutions, yet \( Ax = b \) does not necessarily have a solution for every possible \( b \)?