Worksheet 5

Lay 2.1.4, 2.1.7, 2.1.12, 2.1.23, 2.2.1, 2.2.5, 2.2.18, 2.2.24

2.1.4 Compute $A - 5I_3$ and $(5I_3)A$, where

$$A = \begin{bmatrix} 5 & -1 & 3 \\ -4 & 3 & -6 \\ -3 & 1 & 2 \end{bmatrix}$$

2.1.7 If a matrix is $5 \times 3$ and the product $AB$ is $5 \times 7$, what is the size of $B$?

2.2.18 Solve the equation $AB = BC$ for $A$ assuming that $A$, $B$, and $C$ are square and $B$ is invertible.

2.1.12 Let $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$. Construct a $2 \times 2$ matrix $B$ such that $AB$ is the zero matrix. Use two different nonzero columns for $B$.

2.1.23 Suppose $CA = I_n$ (the $n \times n$ identity matrix). Show that the equation $Ax = 0$ has only the trivial solution. Explain why $A$ cannot have more columns than rows.

2.2.1 Find the inverse of the matrix

$$\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$$

2.2.5 Use the inverse found in Exercise 1 to solve the system

$$8x_1 + 6x_2 = 2$$
$$5x_1 + 4x_2 = -1$$

Let $A$ and $B$ be $n \times n$ invertible matrices. Is it true that $(A + B)(A - B) = A^2 - B^2$? Why or why not?

2.2.19 Solve the equation $C^{-1}(A + X)B^{-1} = I_n$ for $X$ assuming $A, B, C$ are all $n \times n$

2.2.24 Suppose $A$ is $n \times n$ and the equation $Ax = b$ has a solution for each $b \in \mathbb{R}^n$. Explain why $A$ must be invertible. [Hint: Is $A$ row equivalent to $I_n$?]
You’ve seen the properties one-to-one and onto defined for linear transformations. However, we can also define them for any function, linear or not. Recall that a function $f : U \to V$ is onto if for every $v \in V$, we can find at least one $u \in U$ such that $f(u) = v$, and that $f$ is one-to-one for every $v \in V$ we can find at most one $u \in U$ such that $f(u) = v$.

Find ordinary, real valued functions (not necessarily linear) which are both one-to-one and onto, one-to-one but not onto, onto but not one-to-one, and neither onto nor one-to-one. Which of these are linear transformations? Is it possible to find a linear transformation from $\mathbb{R}$ to $\mathbb{R}$ of each type? Why or why not?