Worksheet 5

Lay 2.1.4, 2.1.7, 2.1.12, 2.1.23, 2.2.1, 2.2.5, 2.2.18, 2.2.24

2.1.4 Compute $A - 5I_3$ and $(5I_3)A$, where

$$A = \begin{bmatrix} 5 & -1 & 3 \\ -4 & 3 & -6 \\ -3 & 1 & 2 \end{bmatrix}$$

2.1.7 If a matrix is 5×3 and the product AB is 5×7, what is the size of B?

2.2.18 Solve the equation $AB = BC$ for A assuming that A, B, and C are square and B is invertible.

2.1.12 Let $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$. Construct a 2×2 matrix B such that AB is the zero matrix. Use two different nonzero columns for B.

2.1.23 Suppose $CA = I_n$ (the $n \times n$ identity matrix). Show that the equation $Ax = 0$ has only the trivial solution. Explain why A cannot have more columns than rows.

2.2.1 Find the inverse of the matrix

$$\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$$

2.2.5 Use the inverse found in Exercise 1 to solve the system

$$\begin{aligned} 8x_1 + 6x_2 &= 2 \\ 5x_1 + 4x_2 &= -1 \end{aligned}$$

Let A and B be $n \times n$ invertible matrices. is it true that $(A+B)(A-B) = A^2 - B^2$? Why or why not?

2.2.19 Solve the equation $C^{-1}(A+X)B^{-1} = I_n$ for X assuming A, B, C are all $n \times n$

2.2.24 Suppose A is $n \times n$ and the equation $A\mathbf{x} = \mathbf{b}$ has a solution for each $b \in \mathbb{R}^n$. Explain why A must be invertible. [Hint: Is A row equivalent to I_n?]

You've seen the properties *one-to-one* and *onto* defined for linear transformations. However, we can also define them for any function, linear or not. Recall that a function $f : U \to V$ is onto if for every $v \in V$, we can find *at least* one $u \in U$ such that $f(u) = v$, and that f is one-to-one for every $v \in V$ we can find *at most* one $u \in U$ such that $f(u) = v$.
Find ordinary, real valued functions (not necessarily linear) which are both one-to-one and onto, one-to-one but not onto, onto but not one-to-one, and neither onto nor one-to-one. Which of these are linear transformations? Is it possible to find a linear transformation from $\mathbb{R}$ to $\mathbb{R}$ of each type? Why or why not?