Worksheet 7

4.3.9 Find bases for the null space and column space of the matrix

$$\begin{bmatrix} 1 & 0 & -2 & -2 \\ 0 & 1 & 1 & 4 \\ 3 & -1 & -7 & 3 \end{bmatrix}.$$

What are their dimensions, and what spaces do they live in? (I.e. the null space is a m-dimensional subspace of $\mathbb{R}^n$, for what m and n?).

4.3.25 Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and let H be the set of vectors in $\mathbb{R}^3$ whose second and third entries are equal. Then every vector in H has a unique expression as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, because

$$\begin{bmatrix} s \\ t \\ t \end{bmatrix} = s \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + (t-s) \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

for any s and t. Is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ a basis for H? Why or why not?

a For each of the following sets of polynomials, do they form a basis for $\mathbb{P}_2$? Justify your answer. If possible, write the polynomial $1 + t + t^2$ in terms of each basis.

(a) $S = \{1 + t^2, 1 - t^2\}$

(b) $S = \{t, 2t - 1, t + 3\}$

(c) $S = \{t, 2t - 1, t^2 + t + 1\}$

(d) $S = \{2t^2 + 5t - 1, -t^2 - 3t + 1, -t + 1\}$

b Let A and B be matrices. If $\text{Col}A \subseteq \text{Nul}B$ ($\subseteq$ means "is a subset of"), what is $\text{Nul}BA$?

c Suppose you have a basis $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ of $\mathbb{R}^3$, and suppose $\mathbf{w} = a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$. If you know a, b, c, how can you obtain $\mathbf{w}$? If you know $\mathbf{w}$, how can you obtain a, b, c? (Try to view this as a matrix multiplication).

d Suppose there is a 2×3 matrix C such that

$$C \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = e_1 \qquad C \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} = e_2.$$

Find a 3×2 matrix D such that $CD = I_2$. Is D unique? If you know

$$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

is in the null-space of C, can you find two different matrices D which suffice?