Worksheet 8

4.5.12, 4.6.1, 5.1.2, 5.1.12, 5.1.21b, 5.1.22b, 5.1.26

4.5.12 Find the dimension of the subspace spanned by the vectors:

$$\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -6 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 5 \end{bmatrix}.$$

4.6.1 The matrix

$$A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & 1 & -\frac{5}{2} & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

What are rank A and dim Nul A? Find bases for Col A and Nul A.

Is

$$T(at^3 + bt^2 + ct + d) = (a - c + 5d)t^2 + (-2b + 5c - 6d)$$

a linear transformation? If so, find a basis for its kernel.

Write down a clear and correct definition of the kernel and the range of linear transformation.
Draw a figure illustrating the relationship between the kernel and range, and explain it to somebody near you. (This is deliberately open-ended).

4.7.1
Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be bases for a vector space V, and suppose $\mathbf{b}_1 = 6\mathbf{c}_1 - 2\mathbf{c}_2$ and $\mathbf{b}_2 = 9\mathbf{c}_1 - 4\mathbf{c}_2$.

a. Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$.

b. Find $[\mathbf{x}]_{\mathcal{C}}$ for $\mathbf{x} = -3\mathbf{b}_1 + 2\mathbf{b}_2$. Use part (a).

c. Find the change-of-coordinates matrix from $\mathcal{C}$ to $\mathcal{B}$.

Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$, $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$, and $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2\}$ be bases for a two dimensional vector space. Write an equation that relates the change of basis matrices $\mathcal{C} \overset{P}{\leftarrow} \mathcal{B}$, $\mathcal{D} \overset{P}{\leftarrow} \mathcal{C}$, and $\mathcal{D} \overset{P}{\leftarrow} \mathcal{B}$. Justify your result.