2. \(T(c(x, y)) = T(cx, cy) = (cx + cy, 2cy) = c(x + y, 2y) = cT(x, y)\)

\(T((x, y) + (z, t)) = T(x + z, y + t) = (x + z + y + t, 2(y + t)) = (x + y + z + t, 2y + 2t) = (x + y, 2y) + (z + t, 2t) = T(x, y) + T(z, t)\)

This is a linear transformation.

4. \(T(c(x, y)) = T(cx, cy) = (cx, 2cy, cx + cy) \neq cT(x, y) = (cx, 2c, ex + cy)\)

\(T((x, y) + (z, t)) = T(x + z, y + t) = (x + z, 2x + z + y + t) \neq T(x, y) + T(z, t) = (x, 2x + y) + (z, 2z + t) = (x + z, 4x + y + z + t)\)

This is not a linear transformation.

6. \(T(c(x, y)) = T(cx, cy) = (cx, 1, cy, 1) \neq cT(x, y) = (cx, c, cy, c)\)

\(T((x, y) + (z, t)) = T(x + z, y + t) = (x + z, 1, y + t, 1) \neq T(x, y) + T(z, t) = (x, 1, y, 1) + (z, 1, t, 1) = (x + z, 2, y + t, 2)\)

This is not a linear transformation.

32. \(T(1, 0) = (0, -1)\) and \(T(0, 1) = (1, 0)\). So the standard matrix associated with this linear transformation is 
\[
\begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}
\]

34. \(T(1, 0) = (1, 1, 0)\) and \(T(0, 1) = (2, -2, 1)\). So the standard matrix associated with this linear transformation is 
\[
\begin{pmatrix}
1 & 2 \\
1 & -2 \\
0 & 1
\end{pmatrix}
\]

40. \(T(1, 0) = (1, 1)\) and \(T(x, y) = (x + y, 1, y + 1)\) implies that \(x = 1\) and \(y = 2\). So \((1, 2)\) is mapped to \((3, 1)\).

42. \(T(1, 2) = (1, 5)\) and \(T(x, y) = (x, x + 2y) = (1, 3)\) implies that \(x = 1\) and \(y = 1\). So \((1, 1)\) is mapped to \((1, 3)\).

47. \[
\begin{pmatrix}
1 & -1 \\
2 & 1
\end{pmatrix}
\begin{pmatrix}
0 \\
0
\end{pmatrix}
= \begin{pmatrix}
0 \\
0
\end{pmatrix}
\]
\[
\begin{pmatrix}
1 & -1 \\
2 & 1
\end{pmatrix}
\begin{pmatrix}
1 \\
1
\end{pmatrix}
= \begin{pmatrix}
1 \\
2
\end{pmatrix}
\]
\[
\begin{pmatrix}
1 & -1 \\
2 & 1
\end{pmatrix}
\begin{pmatrix}
0 \\
1
\end{pmatrix}
= \begin{pmatrix}
-1 \\
1
\end{pmatrix}
\]
\[
\begin{pmatrix}
1 & -1 \\
2 & 1
\end{pmatrix}
\begin{pmatrix}
1 \\
1
\end{pmatrix}
= \begin{pmatrix}
0 \\
3
\end{pmatrix}
\]

The area of the quadrilateral with vertices at \((0, 0), (1, 2), (-1, 1), (0, 3)\)
is 3 (draw a picture and calculate the area as that of two triangles) and the area of the square with vertices (0, 0), (1, 0), (0, 1), (1, 1) is 1.

50. The determinant of $A$ is 3. The area of the square in 47. was multiplied by 3, the determinant of $A$. 