Math 5A: Homework #3 Solution

2. $y'' + y' + y = 0$
   The characteristic equation is $r^2 + r + 1 = 0$. Using the quadratic formula, we get that the roots are $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$. Hence the general solution is $y(t) = e^{-\frac{t}{2}}(c_1 \cos \frac{\sqrt{3}}{2}t + c_2 \sin \frac{\sqrt{3}}{2}t)$.

4. $y'' + 2y' + 8y = 0$
   The characteristic equation is $r^2 + 2r + 8 = 0$. Using the quadratic formula, we get that the roots are $-1 + \sqrt{7}i$ and $-1 - \sqrt{7}i$. Hence the general solution is $y(t) = e^{-t}(c_1 \cos \sqrt{7}t + c_2 \sin \sqrt{7}t)$.

6. $y'' - 4y' + 7y = 0$
   The characteristic equation is $r^2 - 4r + 7 = 0$. Using the quadratic formula, we get that the roots are $2 + \sqrt{3}i$ and $2 - \sqrt{3}i$. Hence the general solution is $y(t) = e^{2t}(c_1 \cos \sqrt{3}t + c_2 \sin \sqrt{3}t)$.

8. $3y'' + 4y' + 9y = 0$
   The characteristic equation is $3r^2 + 4r + 9 = 0$. Using the quadratic formula, we get that the roots are $-\frac{2}{3} + \frac{\sqrt{23}}{3}i$ and $-\frac{2}{3} - \frac{\sqrt{23}}{3}i$. Hence the general solution is $y(t) = e^{-\frac{2t}{3}}(c_1 \cos \frac{\sqrt{23}}{3}t + c_2 \sin \frac{\sqrt{23}}{3}t)$.

10. $y'' + y' + 2y = 0$
    The characteristic equation is $r^2 + r + 2 = 0$. Using the quadratic formula, we get that the roots are $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$. Hence the general solution is $y(t) = e^{-\frac{t}{2}}(c_1 \cos \frac{\sqrt{3}}{2}t + c_2 \sin \frac{\sqrt{3}}{2}t)$.

12. $y'' - 4y' + 13y = 0$, $y(0) = 1$, $y'(0) = 0$
    The characteristic equation is $r^2 - 4r + 13 = 0$. Using the quadratic formula, we get that the roots are $2 + 3i$ and $2 - 3i$. Hence the general solution is $y(t) = e^{2t}(c_1 \cos 3t + c_2 \sin 3t)$. Using the first initial condition, we have $y(0) = c_1 = 1$. Using this and taking the derivative of the general solution, we get $y'(t) = 2e^{2t}(\cos 3t + c_2 \sin 3t) + e^{2t}(-3 \sin 3t + 3c_2 \cos 3t)$. Using the second initial condition, we get $y'(0) = 2 + 3c_2 = 0$, so $c_2 = -\frac{2}{3}$. Hence $y(t) = e^{2t}(\cos 3t - \frac{2}{3} \sin 3t)$.

14. $y'' - y' + y = 0$, $y(0) = 0$, $y'(0) = 1$
    The characteristic equation is $r^2 - r + 1 = 0$. Using the quadratic formula, we get that the roots are $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $\frac{1}{2} - \frac{\sqrt{3}}{2}i$. Hence the general solution
is \( y(t) = e^{\frac{t}{2}} (c_1 \cos \sqrt{3}t + c_2 \sin \sqrt{3}t) \). Using the first initial condition, we have \( y(0) = c_1 = 0 \). Using this and taking the derivative of the general solution, we get \( y'(t) = 2e^{t/2}c_2 \sin \sqrt{3}t + e^{t/2}\sqrt{3}c_2 \cos \sqrt{3}t \). Using the second initial condition, we get \( y'(0) = \frac{\sqrt{3}}{2}c_2 = 1 \), so \( c_2 = \frac{2}{\sqrt{3}} \). Hence \( y(t) = \frac{2}{\sqrt{3}} e^{t/2} \sin \sqrt{3}t \).

16. \( y'' + 2y' + 5y = 0 \), \( y(0) = 1 \), \( y'(0) = -1 \)

The characteristic equation is \( r^2 + 2r + 5 = 0 \). Using the quadratic formula, we get that the roots are \(-1 + 2i\) and \(-1 - 2i\). Hence the general solution is \( y(t) = e^{-t}(c_1 \cos 2t + c_2 \sin 2t) \). Using the first initial condition, we have \( y(0) = c_1 = 1 \). Using this and taking the derivative of the general solution, we get \( y'(t) = -e^{-t}(\cos 2t + c_2 \sin 2t) + e^{-t}(-2 \sin 2t + 2c_2 \cos 2t) \). Using the second initial condition, we get \( y'(0) = -1 + 2c_2 = -1 \), so \( c_2 = 0 \). Hence \( y(t) = e^{-t} \cos 2t \).