Practice Final – Math 3B, Fall 2004

The questions in the exam will be similar in nature to the following:

1. Evaluate the limit
   \[
   \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \left( 4 + \frac{i}{n} \right)^4
   \]

2. Evaluate the limit
   \[
   \lim_{h \to 0} \frac{1}{h} \int_{2}^{2+h} x^3 \log(x + 1) \, dx
   \]

3. Find the derivative of the function
   \[
   g(x) = \int_{2x}^{4x} \frac{t^2 - 1}{2 + \cos(t)} \, dt
   \]

4. Find the area of the region enclosed by the curves \( y = \sqrt{x + 1}, y = \frac{1}{1+x}, \) the y-axis, and \( x = 1. \)

5. Find the volume of the solid obtained by rotating the region bounded by the curves \( y = \sqrt{x^2 + 1}, y = x^2, x = 0, \) and \( x = 1, \) around the \( OX \) axis.

6. Evaluate the integral
   \[
   \int \cos(x) \ln(\sin(x)) \, dx
   \]

7. Evaluate the integral
   \[
   \int e^x \sin(x) \, dx
   \]

8. Use integration by parts to show that
   \[
   \int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx
   \]

9. Evaluate the integral
   \[
   \int \cos^5 x \sin^2 x \, dx
   \]

10. Evaluate the integral
    \[
    \int \tan^3 x \sec^5 x \, dx
    \]
11. Find
\[ \int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2 + 9)^{3/2}} \, dx \]

12. Evaluate the integral
\[ \int \frac{x^2}{(4 - x^2)^{3/2}} \, dx \]

13. Evaluate
\[ \int \frac{x^2 + 4}{x(x^2 + 1)} \, dx \]

14. Evaluate
\[ \int \frac{e^{2x}}{e^{2x} + 3e^x + 2} \, dx \]

15. Determine whether the following integral is convergent or divergent:
\[ \int_0^\infty \frac{1}{\sqrt{x}(1 + x)} \, dx \]

16. Determine whether the following integral is convergent or divergent:
\[ \int \frac{1}{\sqrt{x} \sin x} \, dx \]

17. Evaluate
\[ \int_{-\infty}^{\infty} \frac{e^{2x}}{e^{4x} + 2e^{2x} + 1} \, dx \]

18. Compute the length of the curve
\[ y = \cosh x, \quad 0 \leq x \leq 1 \]

19. Compute the length of the curve
\[ y = 1 + 6x^{3/2}, \quad 0 \leq x \leq 1 \]

20. Find the area of the surface obtained by rotating the curve
\[ y = \cosh x, \quad 0 \leq x \leq 1 \]

about the x-axis.