Practice Final

1. (a) If \( f(x) = \sin(x^2 - 1) \), find \( f'(x) \).
   (b) If \( g(x) = \ln(x^3) \), find \( g'(x) \).

2. Let \( f(x) = \frac{x^2 + 2x}{|x|} \)
   (a) Find \( \lim_{x \to 0^+} f(x) \)
   (b) Find \( \lim_{x \to 0^-} f(x) \)
   (c) Find \( \lim_{x \to 0} f(x) \)

3. Suppose \( V(t) = (t - 1)^2 \) with \( 0 \leq t \leq 10 \) represents the volume of a balloon at time \( t \).
   (a) What is the average rate of change of volume between \( t = 2 \) and \( t = 8 \)?
   (b) What is the instantaneous rate of change of volume at \( t = 3 \)?
   (c) When is the volume changing the fastest?

4. Suppose the position of a particle at time \( t \) is given by \( s(t) = \sqrt{t} \sin t \).
   What is the acceleration of the particle at time \( t = \pi/2 \).

5. Consider the indeterminate form \( \lim_{x \to 0^+} \frac{\ln x}{x - 1} \)
   (a) What type is this indeterminate form?
   (b) Compute the limit.

Formulas to keep in mind and additional things to study:

1. \( \frac{d}{dx} \ln x = \frac{1}{x} \)
2. \( \frac{d}{dx} \sin x = \cos x \)
3. \( \frac{d}{dx} \cos x = -\sin x \)
4. \( \frac{d}{dx} f(x)g(x) = f'(x)g(x) + g'(x)f(x) \) Product Rule
5. \( \frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2} \) Quotient Rule

6. Volume of a cone of height \( h \) and base radius \( r \) is \( V = \frac{1}{3} \pi r^2 h \)
7. Look over the last homework assignment. In particular, know the following:
   (a) A function is increasing when \( f'(x) > 0 \)
   (b) A function is decreasing when \( f'(x) < 0 \)
   (c) An odd function is when \( f(-x) = -f(x) \) like \( \sin x \)
   (d) An even function is when \( f(-x) = f(x) \) like \( \cos x \)
(e) Vertical asymptotes happen when the denominator of \( f'(x) \) is zero

(f) Horizontal asymptotes happen when \( \lim_{x \to \infty} f(x) \) is a constant. (or to \(-\infty\))

(g) An inflection point is when \( f''(x) = 0 \)

8. To sketch a function do the following (practice some book examples):

(a) Find out where the function is increasing or decreasing by calculating when \( f' \) is positive or negative.

(b) Find out all the critical points (where \( f'(x) = 0 \) or where \( f'(x) \) doesn’t exist). These are the local mins and maxes (there are horizontal tangent lines at these points).

(c) Figure out if there are any vertical asymptotes.

(d) Calculate \( f''(x) \). Figure out where it is positive or negative. This is when it is concave up and concave down respectively.

(e) A continuous function changes from a concave up region to a concave down region at inflection points (see above).

(f) Figure out what happens as \( x \) goes to positive or negative infinity (are there horizontal or slant asymptotes, or neither?)