Thursday August 16, 2007

Name:________________________

Math 5A, Exam 1

This test is worth 100 points. No notes or calculators are allowed. Good luck!

1. (30 points) Solve the differential equations below:

   (a) $y'' + 3y' - 10y = 0$

   (b) $y'' + 2y' + 10y = 0$

   (c) $y'' + 8y' + 16y = 0$, $y(0) = 2$, $y'(0) = 2$

   (d) $y^{(5)} + y^{(4)} - y''' + 15y'' = 0$

   Hint: The characteristic equation is $r^2(r + 3)(r^2 - 2r + 5) = 0$
2. (20 points) A 4 kg block, resting on a table, is attached to a spring connected to the wall. It takes 300N to move the block 3m from its equilibrium point. Assume that there is no friction between the block and the table. The block is pulled to the right until the spring is stretched 3m and released with a velocity of 20m/s.

(a) Write down a differential equation with initial conditions that describes the motion of the block.

(b) Solve the IVP you have written down in part (a).

(c) How far from its equilibrium point does the block travel? Hint: Your solution in part (b) can be written in the form $x(t) = A \cos(\omega_0 t - \delta)$
3. *(50 points)* Solve the differential equations below:

(a) \( y'' - y' - 2y = 16te^{-2t} \)

(b) \( y'' - 3y' + 2y = 6e^{2t} \)
(c) \[ y'' - 2y' + y = \frac{e^t}{t}, \quad y(1) = e, \quad y'(1) = e \]

(d) \[ y''' - 4y'' + 3y' = 6t - 2 \]