1.6 - 1 b,e
b) Let $m = 1$ and $n = -1$. Then $15(1) + (-1)12 = 3$.
e) Suppose there are integers $m$, $n$, and $t$ such that $15m + 16n = t$. Let $r = 5m$ and $s = 2n$, which are both integers. Then the above equation becomes $3r + 8s = t$.

1.6 - 5 b,e
b) Take any $x \in \mathbb{R}$. Let $y = -x$. Then $x + y = x + (-x) = x - x = 0$.
e) Suppose $a$, $b$, $c$, and $d$ are integers, and $a$ divides $b - c$ and $a$ divides $c - d$. Then $b - c = aq$ for some integer $q$, and $c - d = ar$ for some integer $r$. So $d = c - ar$. Therefore, $b - d = b - (c - ar) = b - c + ar = aq + ar = a(q + r)$. And $q + r$ is an integer, so $a$ divides $b - d$.

1.6 - 8 c
c) In this problem, the dude says that $a$ divides $b$, so $a = bk$ for some integer $k$. This is wrong. If $a$ divides $b$, then $ak = b$ for some integer $k$. So then $(ak)^n = a^n k^n = b^n$ so since $k^n$ is an integer, $a^n$ divides $b^n$. I would give this proof a C. Small error, proof mostly right.

2.1 - 1 b,e
b) $\{x \in \mathbb{Z} : x < 17\}$
e) $\{x \in \mathbb{R} : -5 \leq x \leq -1\}$

2.1 - 6
a) $\mathcal{P}(X) = \{\emptyset, \{0\}, \{\square\}, \{\Delta\}, \{0, \Delta\}, \{0, \square\}, \{\Delta, \square\}, X\}$
b) $\mathcal{P}(X) = \{\emptyset, X, \{S\}, \{\{S\}\}\}$
c) $\mathcal{P}(X) = \{\emptyset, X, \emptyset, \emptyset, \{\{a\}\}, \{\{b\}\}, \{\{a, b\}\}, \emptyset, \{\emptyset\}, \{\emptyset, \{a\}\}, \{\emptyset, \{b\}\}, \{\emptyset, \{a, b\}\}, \{\emptyset, \{a\}\}, \{\emptyset, \{b\}\}, \{\emptyset, \{a, b\}\}\}$
d) $\mathcal{P}(X) = \{\emptyset, X, \{1\}, \{\{2, \{3\}\}\}\}$
e) $\mathcal{P}(X) = \{\emptyset, X, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$

2.1 - 8 b, c
b) If \( A = B \), then \( A \subseteq B \), and \( \mathcal{P}(B) \subseteq \mathcal{P}(A) \). So for instance, let \( A = B = \emptyset \), then \( \mathcal{P}(A) = \mathcal{P}(B) = \emptyset \).

c) The smallest possible set is the empty set, whose power set is \( \{ \emptyset \} \) (i.e. a one element set, not the empty set). So this is not possible.

2.2 - 1 b,d,f,h,j

b) \( A \cap B = \emptyset \)

d) \( B - C = \{0, 6\} \) so \( A - (B - C) = \{1, 3, 5, 7, 9\} \)

f) \( C \cap D = \{1, 2, 5, 7, 8\} \) so \( A \cap (C \cap D) = \{1, 5, 7\} \)

h) \( B \cup C = \{0, 1, 2, 4, 5, 6, 7, 8\} \) so \( A \cap (B \cup C) = \{1, 5, 7\} \)

j) \( A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \) and \( C \cap D = \{1, 2, 5, 7, 8\} \), so \( (A \cup B) - (C \cap D) = \{0, 3, 4, 6, 9\} \)

2.2 - 13 b

b) Take any \( S \in \mathcal{P}(A) \cup \mathcal{P}(B) \). Then \( S \in \mathcal{P}(A) \) or \( S \in \mathcal{P}(B) \) (or both). Then \( S \subseteq A \) or \( S \subseteq B \). So \( S \subseteq (A \cup B) \). Therefore \( S \in \mathcal{P}(A \cup B) \). Thus, \( \mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B) \).

c) If \( A \) and \( B \) are disjoint (i.e. \( A \cap B = \emptyset \)), then equality doesn’t hold. Try \( A = 1 \) and \( B = 2 \). If \( A \subseteq B \), then the equality will hold since then \( \mathcal{P}(A \cup B) = \mathcal{P}(B) \) and \( \mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(B) \).