Math 8: Homework #7 Solution

3.1 - 4 a
a) \( A = \emptyset, \ C = \{1\}, \ D = \emptyset, \ B = \{2\} \)

3.1 - 9 a,c,g
a) \( R \circ S = \{(3,5),(5,2)\} \)

3.1 - 20 b, c, g
b) \((a, c) \notin BXD \Rightarrow a \notin B \lor c \notin D \)

c) \(AXB\) is cartesian product, not multiplication

g) If \((x, z) \in R\) and \((y, z) \in R\), then it is not necessarily true that \(x = y\)

3.2 - 1 d, i, j
d) This is clearly not reflexive \(a < a\) is not true. Also not symmetric since if \(a < b\), then \(b < a\) is false. This is transitive however.

i) This is not reflexive since \(l\) cannot be perpendicular to itself. It is symmetric. It is not transitive.

j) This is not reflexive (try \((0,-1))\). This is symmetric. This is not transitive (try \((0,0), (1,2), (3,2))\).

3.2 - 4 e, j
e) Since \(x^2 + y^2 = x^2 + y^2\), this relation is reflexive. Since \(x^2 + y^2 = a^2 + b^2 \Rightarrow a^2 + b^2 = x^2 + y^2\), then this relation is symmetric. And since if \(x^2 + y^2 = a^2 + b^2\) and \(a^2 + b^2 = c^2 + d^2\), then \(x^2 + y^2 = c^2 + d^2\), this relation is transitive. So therefore this relation is an equivalence relation. The equivalence classes of \((1,2)\) and \((4,0)\) are circles of respective radii \(\sqrt{5}\) and 4 centered at the \((0,0)\).
j) This relation is reflexive since \( f' = f' \). This relation is symmetric because if \( fRg \) then \( f' = g' \), so \( g' = f' \), therefore \( gRf \). This relation is also transitive. If \( fRg \) and \( gRh \), then \( f' = g' \) and \( g' = h' \), so \( f' = h' \), so \( fRh \). Therefore this relation is an equivalence relation. 3 elements from \( x^2/R \) are: \( x^2, x^2 + 1, x^2 = 2 \). 3 elements from \((4x^3 + 10x)/R \) are: \( 4x^3 + 10x + 2, 4x^3 + 10x + 37 \). \( x^3/R \) are functions of the form \( x^3 + c \) where \( c \) is a constant. 7/R are any constant function.

3.2 - 7 c

c) \( \exists p \) such that \( x \equiv_m p \) and \( y \equiv_m p \). Since \( \equiv_m \) is an equivalence relation, then \( p \equiv_m y \) by symmetry. And by transitivity \( x \equiv_m y \). So \( \exists \emptyset = \emptyset \)

3.2 - 8

a) \( R \) is reflexive since for all \( x \in \mathbb{N}, x+x = 2x \) is divisible by 2. Suppose that \( x+y \) is divisible by 2, then clearly \( y+x \) is divisible by 2, so \( R \) is symmetric. If \( x+y \) is divisible by 2 and \( y+z \) is divisible by 2, then \( x+y = 2q \) for some \( q \) and \( y+z = 2r \) for some \( r \). So \( x+z = 2q - y + 2r - y = 2(q + r - y) \). So \( x+z \) is divisible by 2. So \( R \) is transitive. Therefore since \( R \) is reflexive, symmetric, and transitive, it is an equivalence relation.

b) \( S \) is not reflexive, \( 1+1 \) is not divisible by 3. \( S \) is symmetric however since if \( x+y \) is divisible by 3, so is \( y+x \). However, \( S \) is not transitive, since \( 1+2 \) is divisible by 3 and \( 2+1 \) is divisible by 3, but \( 1+1 \) is not divisible by 3. So since \( S \) is not reflexive, symmetric, and transitive, it is not an equivalence relation.

3.2 13

Since \( R \) is symmetric and transitive on \( A \). Take any \( a \in A \), then there is a \( b \in A \) such that \( aRb \). Now since \( R \) is symmetric, then \( bRa \). And since \( R \) is transitive, then by combining the above, \( aRa \). So \( R \) is reflexive on \( A \).