Paul Erdős

“Problems worthy of attack prove their worth by fighting back.”

Jordan Schettler

UCSB

3/8/2013
Outline

Tragic

Gifted

Inventive

Prolific

Paul Erdős
Tragic
Budapest, Hungary...
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- Two sisters die of scarlet fever in 1913
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- A few days later, brother Paul was born

Figure: Paul, age 8
Budapest, Hungary...

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- “White Terror” in Hungary: Fascism, Anti-Semitism

Figure: Paul, age 8
Budapest, Hungary...

- Two sisters die of scarlet fever in 1913
- A few days later, brother Paul was born
- Father was captured by Russians in WWI
- “White Terror” in Hungary: Fascism, Anti-Semitism
- Mother gets banned from teaching for 26 years

Figure: Paul, age 8
Gifted
As a teenager...

\[ \prod_{p \leq x} p < 4x \] for all \( x > 0 \).

\[ \Rightarrow \] "cleaner" proof of Bertrand's Postulate: Chebyshev said it, we'll say it again: There's always a prime between \( N \) and \( 2N \).

Paul Erdős

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As a teenager...

Erdős proved

\[ \prod_{p \leq x} p < 4^x \]

for all \( x > 0 \).
As a teenager...

Erdős proved

$$\prod_{p \leq x} p < 4^x$$

for all \( x > 0 \).

~~ “cleaner” proof of Bertrand’s Postulate:

*Chebyshev said it, we’ll say it again:*

*There’s always a prime between \( N \) and \( 2N \).*
City Park, Early 1930’s

Paul Erdős

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Erdős left Hungary in 1934

Figure: Paul and Mom Anna
Erdős left Hungary in 1934

- He always remained a very devoted son

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- Ultimately, he had no permanent home & few possessions

Figure: Paul and Mom Anna
Erdős left Hungary in 1934

- He always remained a very devoted son
- Ultimately, he had no permanent home & few possessions
- He ran off of Benzedrine, coffee, and passion

**Figure:** Paul and Mom Anna
Inventive
Ramsey’s Theorem

Any party with 6 people has a group of 3 who all know each other or all don’t know each other.

Paul Erdős
Ramsey’s Theorem

Any party with 6 people has a group of 3 who all know each other or all don’t know each other.

Color edges blue or red in a complete graph of 6 vertices.
Ramsey’s Theorem

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There is always a triangle of some color.
Ramsey’s Theorem

Any party with 6 people has a group of 3 who all know each other or all don’t know each other.

Color edges blue or red in a complete graph of 6 vertices.

There is always a triangle of some color.

A party of 5 is not enough!
Erdős and company proved

Any party with 18 people has a group of 4 who all know each other or all don’t know each other.
Erdős and company proved

Any party with 18 people has a group of 4 who all know each other or all don’t know each other.

Moreover, 17 is not enough!
Let $M_n = 2^n - 1$ (the $n$th Mersenne number)
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$$
\sum_{n=1}^{\infty} \frac{\mu(n)}{M_n}
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$$\sum_{n=1}^{\infty} \frac{\mu(n)}{M_n} = \frac{1}{2}$$
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\[
E = \sum_{n=1}^{\infty} \frac{1}{M_n}
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\[ E = \sum_{n=1}^{\infty} \frac{1}{M_n} = ??, \]
Let $M_n = 2^n - 1$ (the $n$th Mersenne number)

$$\sum_{n=1}^{\infty} \frac{\mu(n)}{M_n} = \frac{1}{2}$$

$$E = \sum_{n=1}^{\infty} \frac{1}{M_n} = \text{??},$$

but, in 1948, Erdős proved $E$ is irrational.
Lambert Series

\[
\sum_{n=1}^{\infty} f(n) \frac{x^n}{1 - x^n} = \sum_{n=1}^{\infty} \left( \sum_{d|n} f(d') \right) x^n
\]
Lambert Series

\[ \sum_{n=1}^{\infty} f(n) \frac{x^n}{1 - x^n} = \sum_{n=1}^{\infty} \left( \sum_{d|n} f(d) \right) x^n \]

In particular, \( x \mapsto 1/2 \) gives

\[ \sum_{n=1}^{\infty} \mu(n) \frac{2^{-n}}{1 - 2^{-n}} = \frac{1}{2} \]
Lambert Series

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\sum_{n=1}^{\infty} f(n) \frac{x^n}{1 - x^n} = \sum_{n=1}^{\infty} \left( \sum_{d|n} f(d) \right) x^n
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In particular, \( x \mapsto 1/2 \) gives

\[
\sum_{n=1}^{\infty} \mu(n) \frac{2^{-n}}{1 - 2^{-n}} = \frac{1}{2}
\]

and

\[
E = \sum_{n=1}^{\infty} \frac{2^{-n}}{1 - 2^{-n}} = \sum_{n=1}^{\infty} \frac{\sigma_0(n)}{2^n}
\]
Proofs from THE BOOK

He was not religious, and jokingly referred to God as the SF (Supreme Fascist), a being who selfishly kept the best proofs of all mathematical theorems in a transfinite BOOK.

Paul Erdős
Erdős to English Dictionary

boss |bɒs|:

The image contains a dictionary entry for the word "boss." The pronunciation is given as "bɒs."
Erdős to English Dictionary

boss |bôs|: a woman.
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Tragic | Gifted | Inventive | Prolific

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Prolific
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<td>▶ He wrote over 1500 papers (most in history, but 2nd to Euler in number of pages)</td>
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Non-Wiki References