How to Mathematize the World: Black Holes, Oil Spills, the Spread of AIDS...

Charles Collingwood and Jordan Schettler

$\subseteq$ Rincon High School $\cap$ University of Arizona

1/22/11

Outline

Introduction

Charles Collingwood

- Teacher of 15 years
- Currently at Rincon High School
- Teaching A.P. Calculus, Precalculus, and Stats
- Also, a grad student at U. of Ariz.

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- Students ONLY learn the mathematical “alphabet” in K-10.
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- We start by analyzing properties of functions abstractly.
- These skills are reinforced with thoughtful labs.
- Carefully written lab reports are required.

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What You Should Take Away

- We are highly motivated and passionate educators.
- We want to share that enthusiasm.
- We'll discuss past, present, and upcoming labs.
- There will be a guided activity which studies income inequality using both integral and differential calculus.

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Definition

math•e•ma•tize |ˈmath(ə)məˌtīz|

verb [trans.]

model and quantify a situation or idea in mathematical terms.

Physics

Galileo: Quadratic Functions

- An object is dropped from 600 ft (no wind).
- The distance traveled $d(t)$ after t seconds is recorded:

t	0	1	2	3	4
$d(t)$	0	32	128	288	512

- Find formulas for $d(t)$ and the height $h(t)$.
- When does the object hit the ground?
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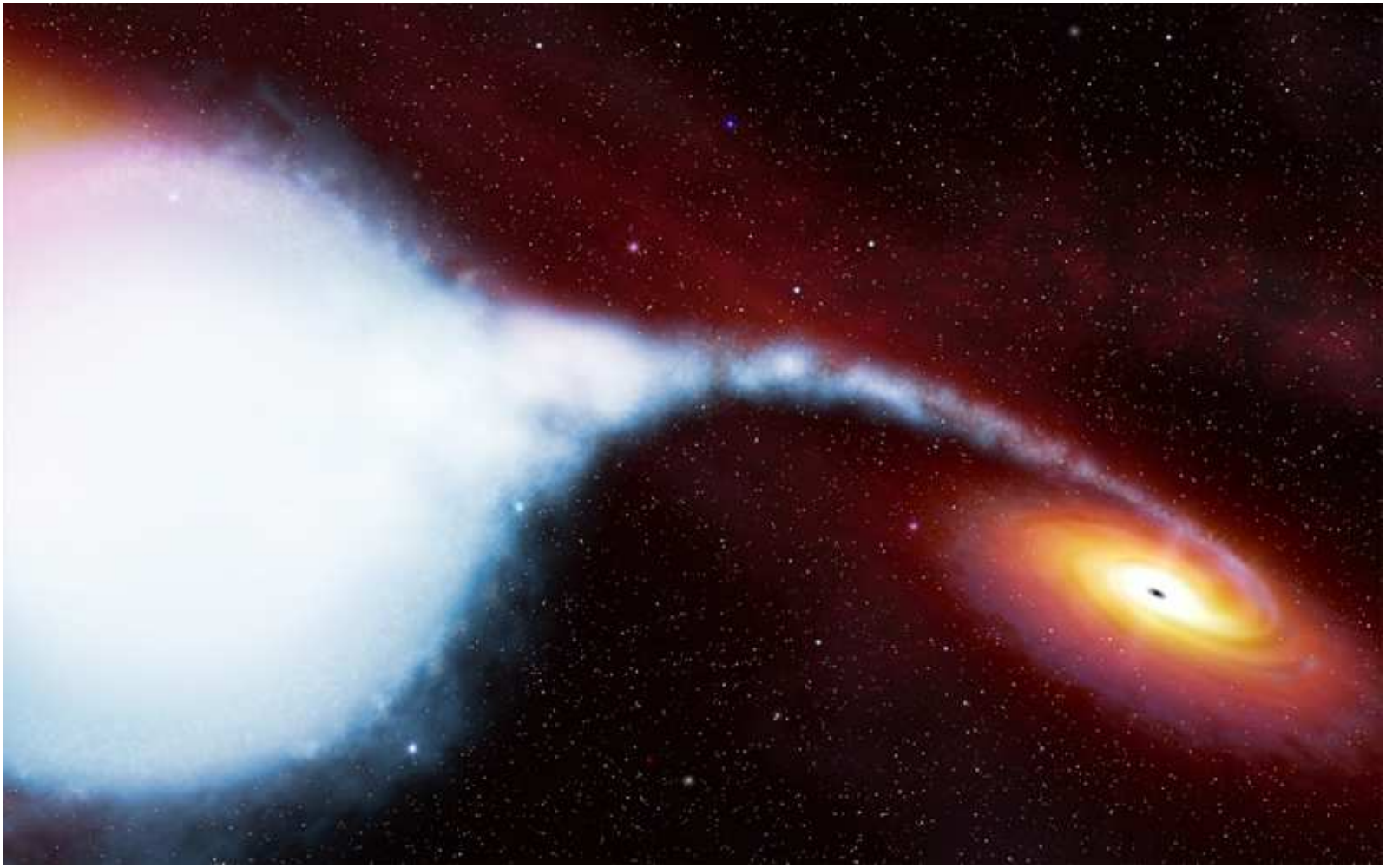

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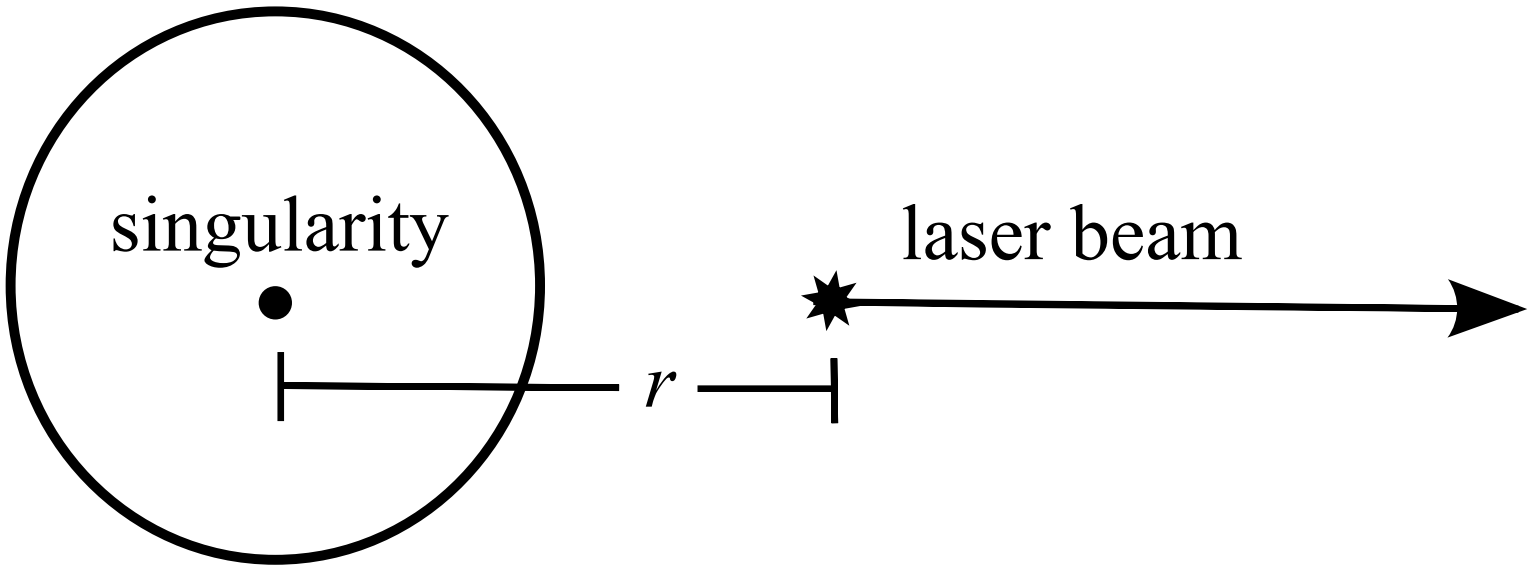

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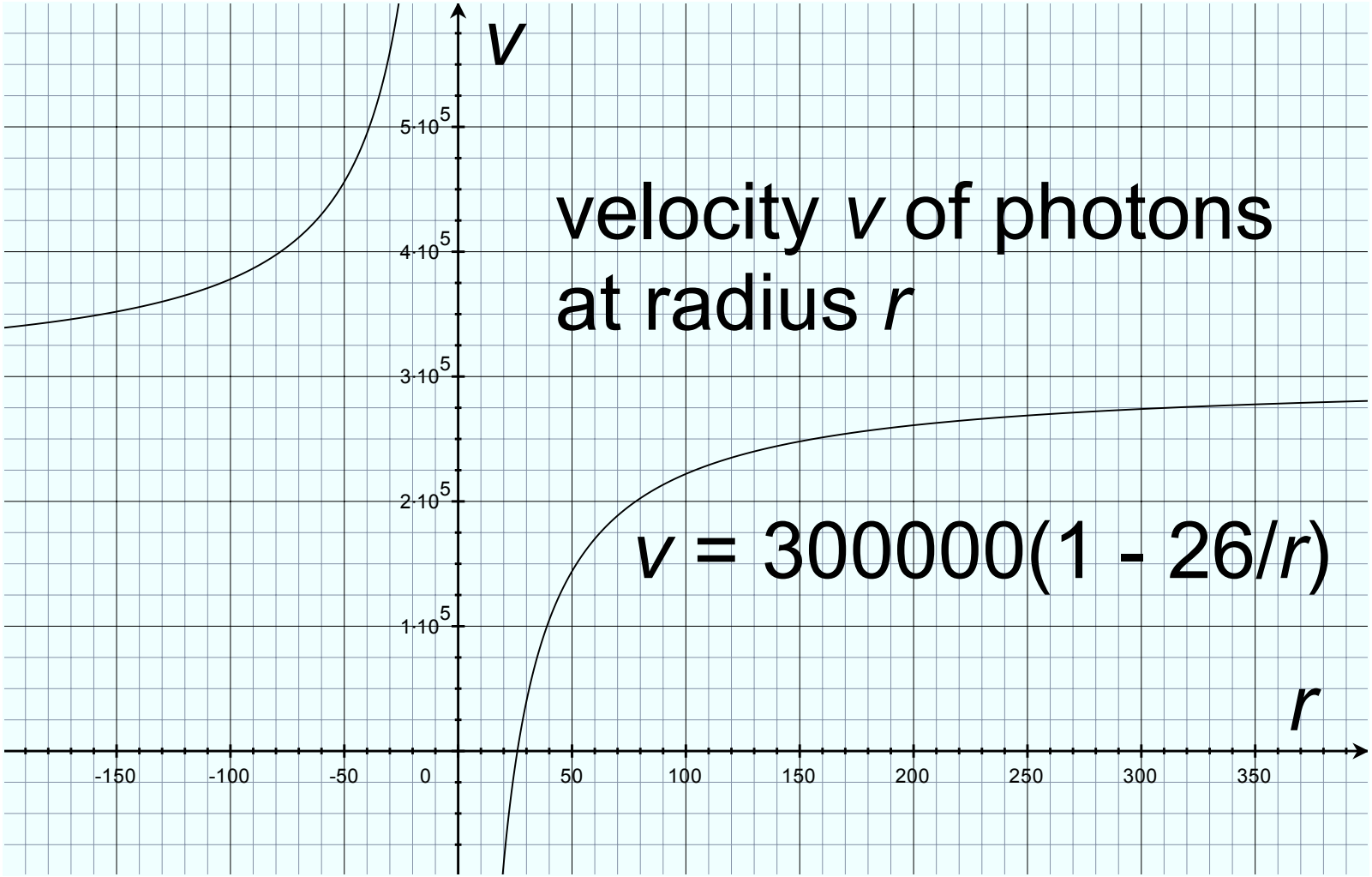
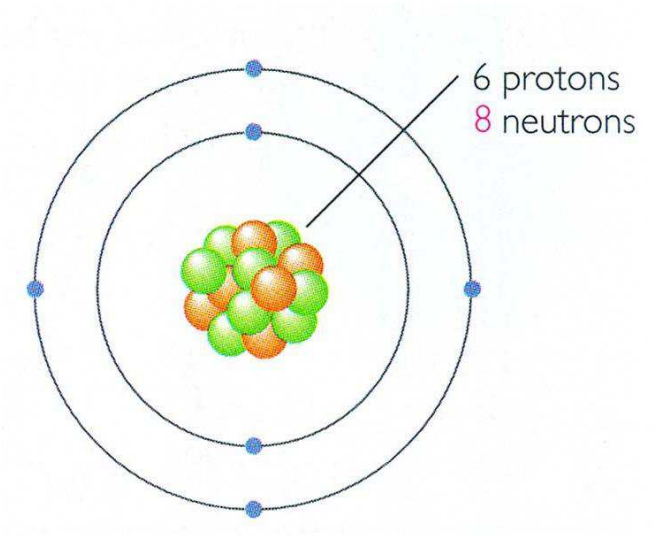


Carbon-14: Exp/Log Functions

- Carbon-14 has a half-life $\lambda = 5,730$ years.



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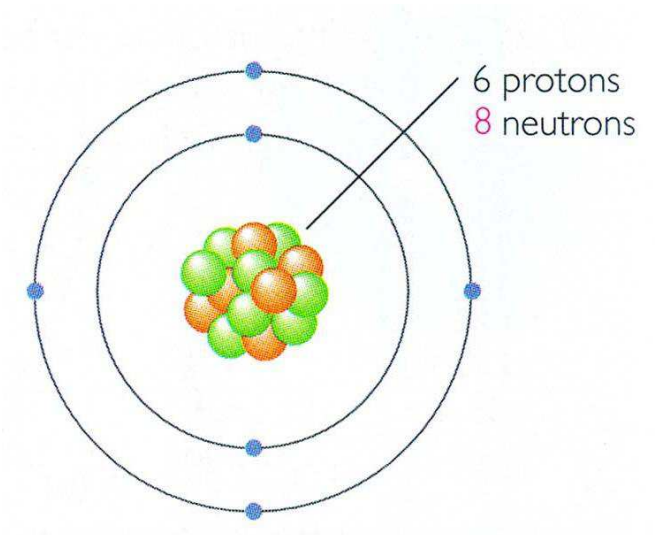

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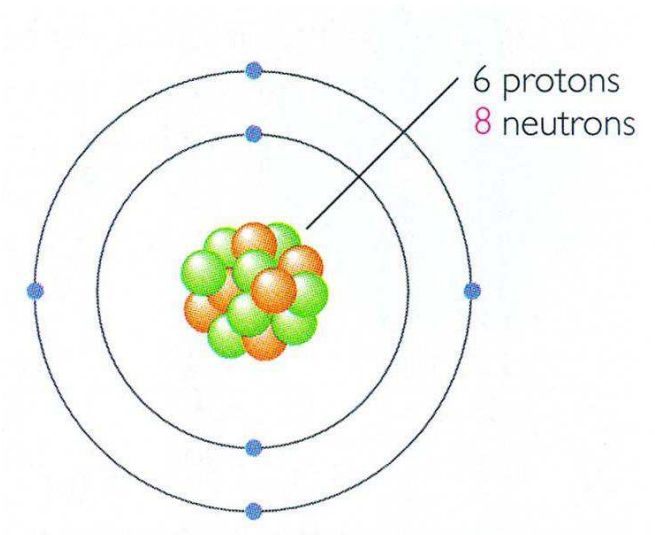

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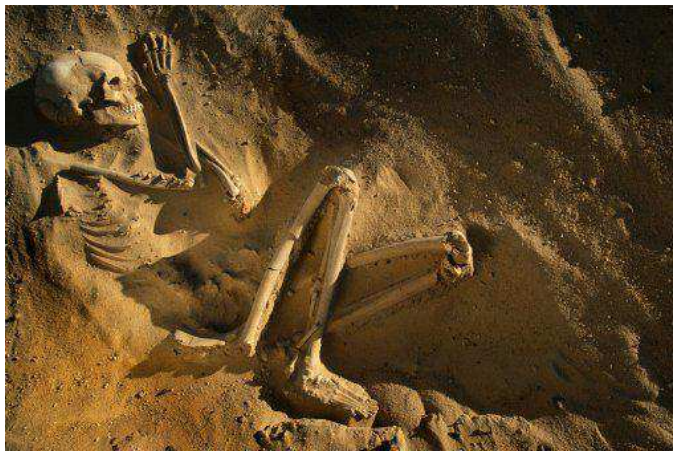

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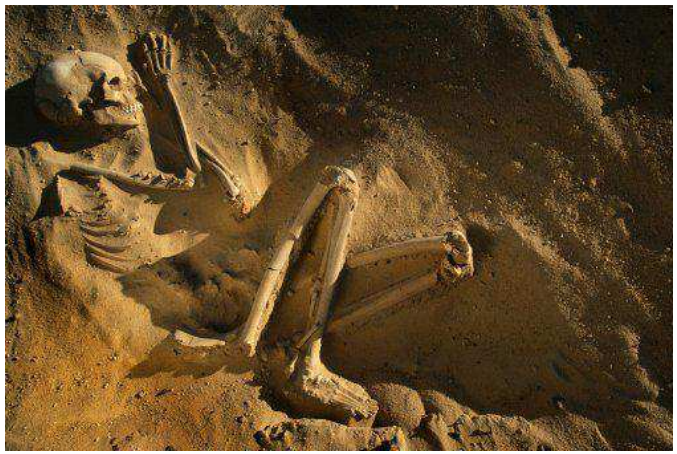

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Environmental Issues

The BP Oil Spill: Inter/Extra-polation

- April 20, 2010: Deepwater Horizon explosion

- How much will it cost BP for the clean-up?

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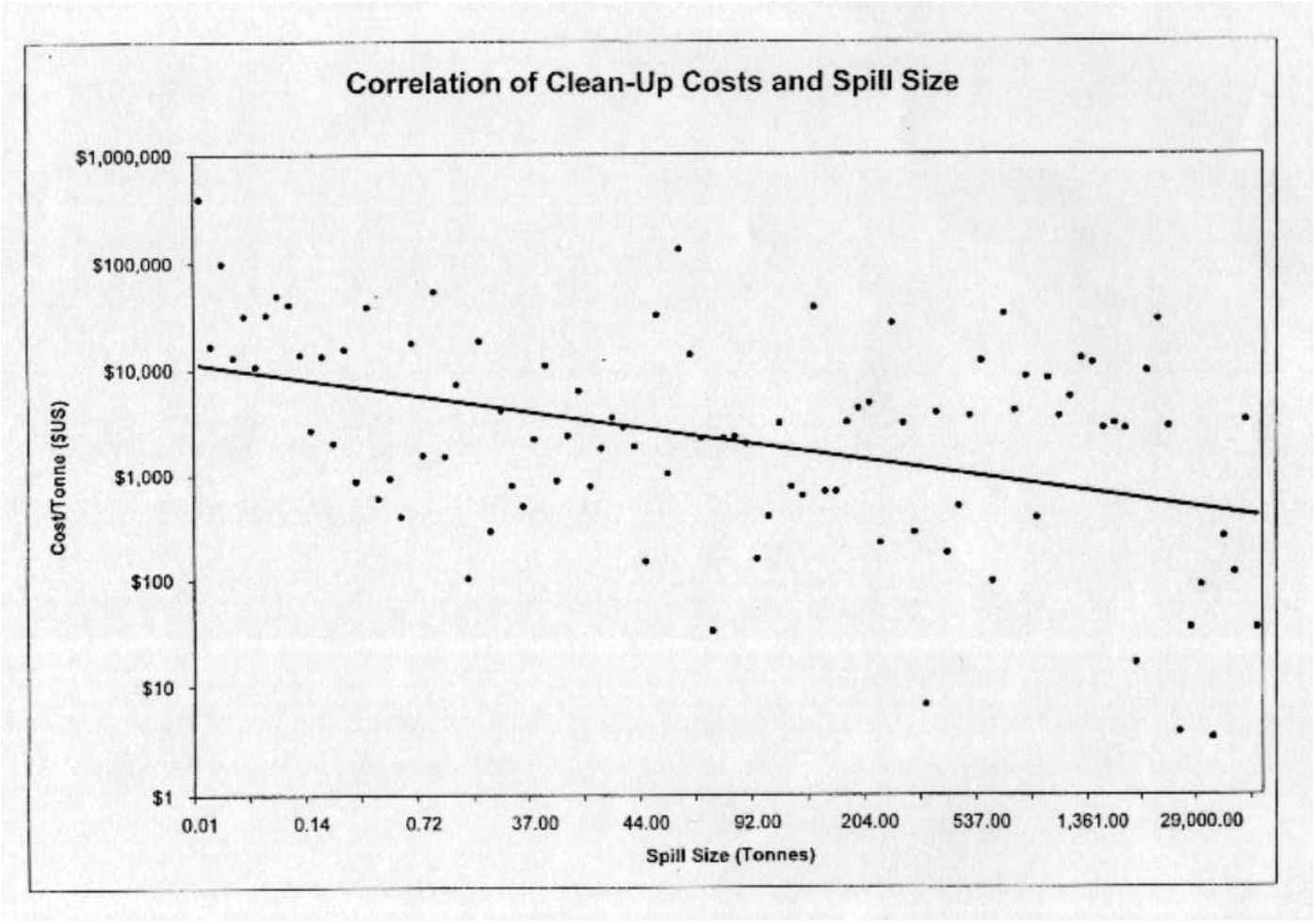


There are many confounding variables! When will BP recoup?

Dyson AirBlade: Linear Functions/Modeling

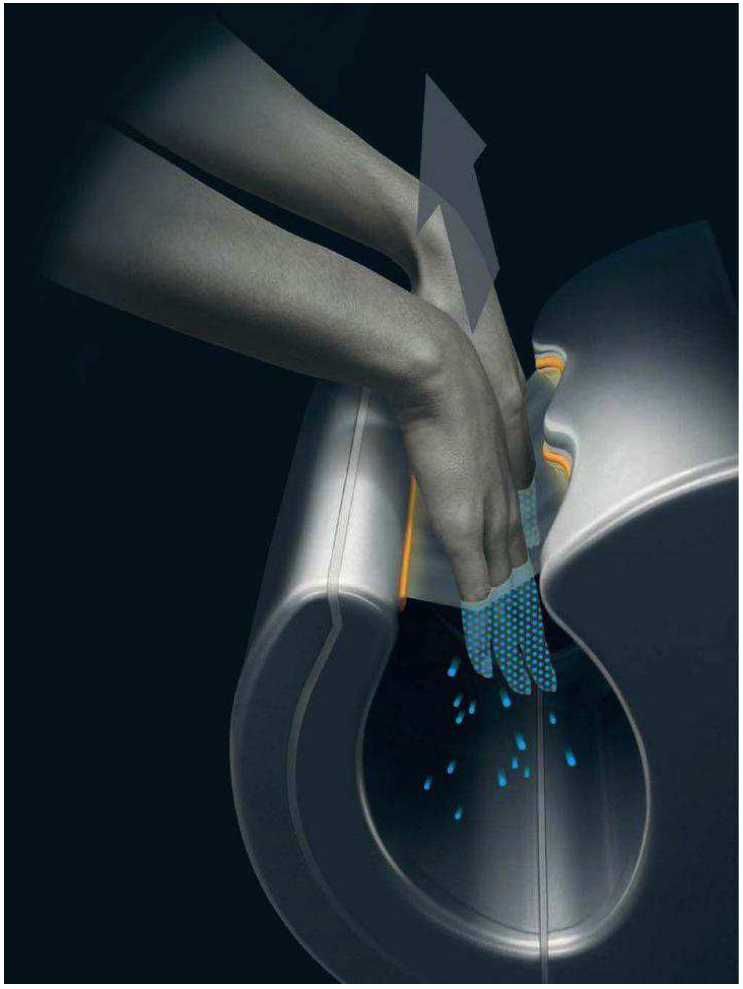
$\nexists$soap

$\nexists$paper towels

$\nexists$urinal

$\nexists$window

Dyson AirBlade: Linear Functions/Modeling



- Could Rincon get a $1,200 Dyson AirBlade?
- Linear cost models: paper towels vs hand dryer
- Where's the intersection point?
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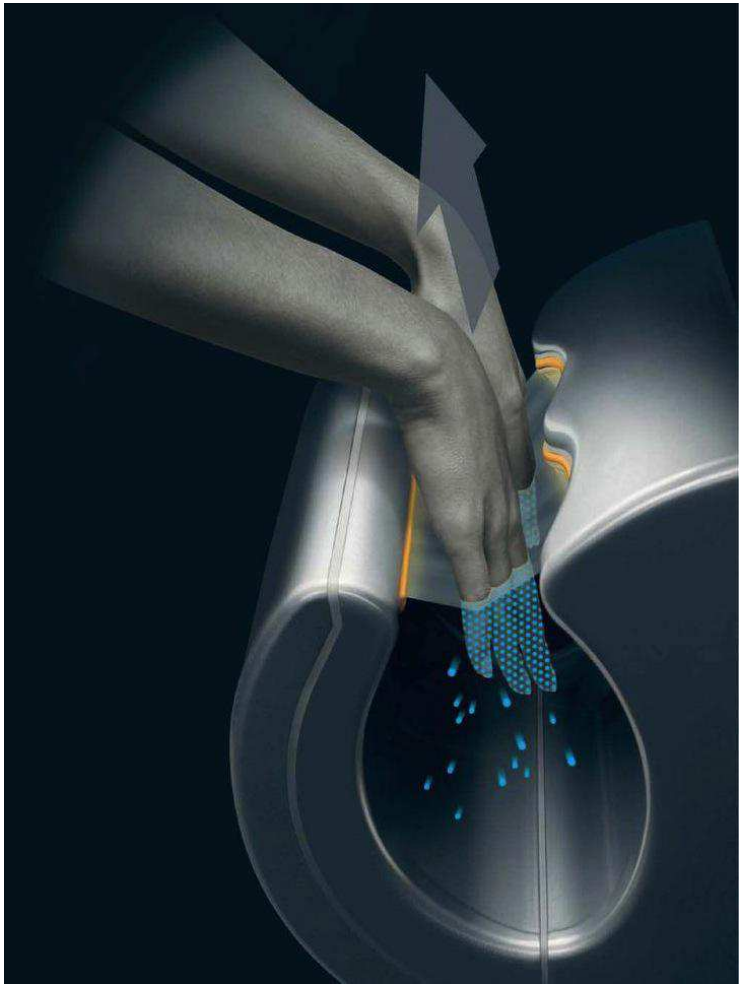

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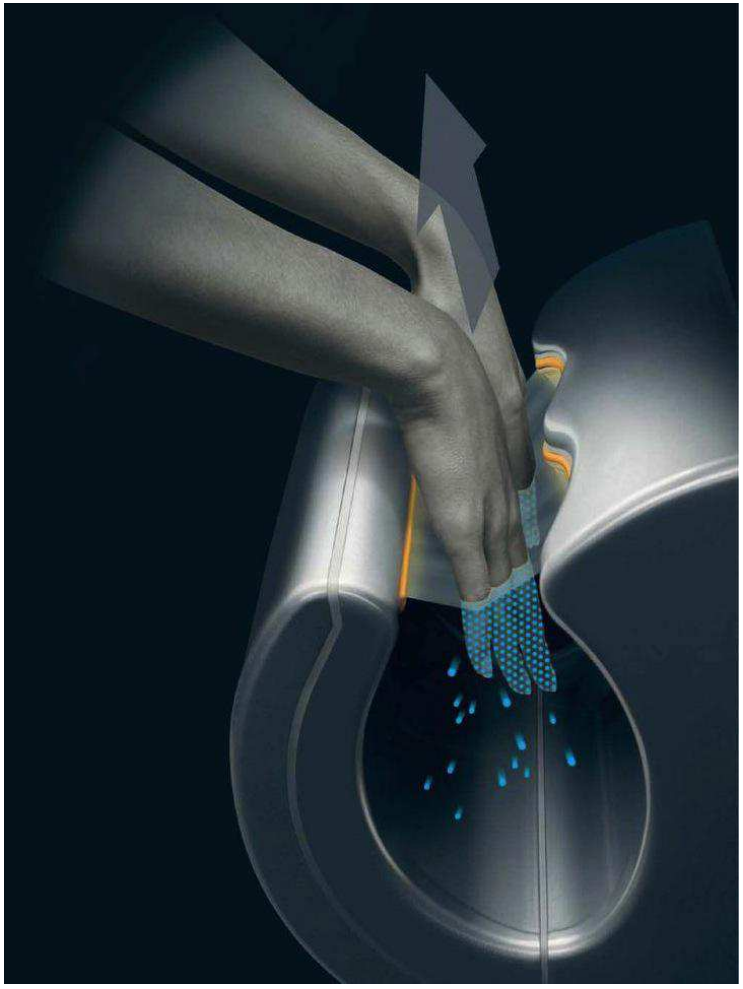

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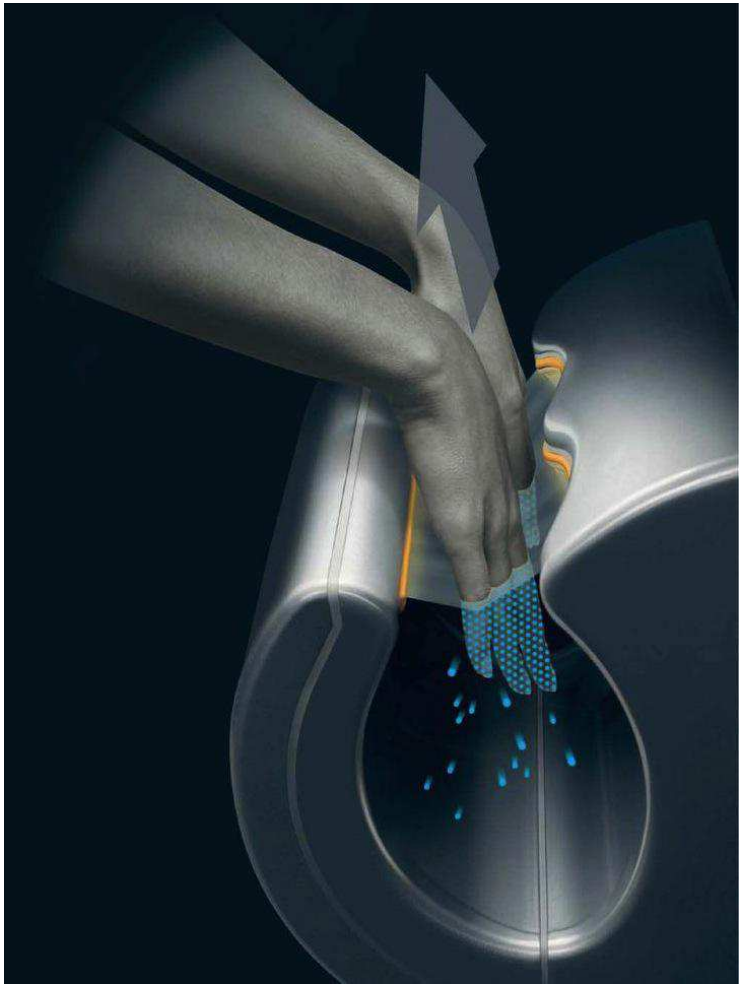

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Social Justice

Underrepresentation: Binomial Distribution

- There's been 4 women out of 112 justices.
- The ratio of men to women is roughly 1 to 1.
- Let $p = 0.5$ and $n = 112$:

$$P(k \leq 4) = 1.242 \times 10^{-27}$$

- 47 is minimal such that

$$P(k \leq 47) > 0.05$$

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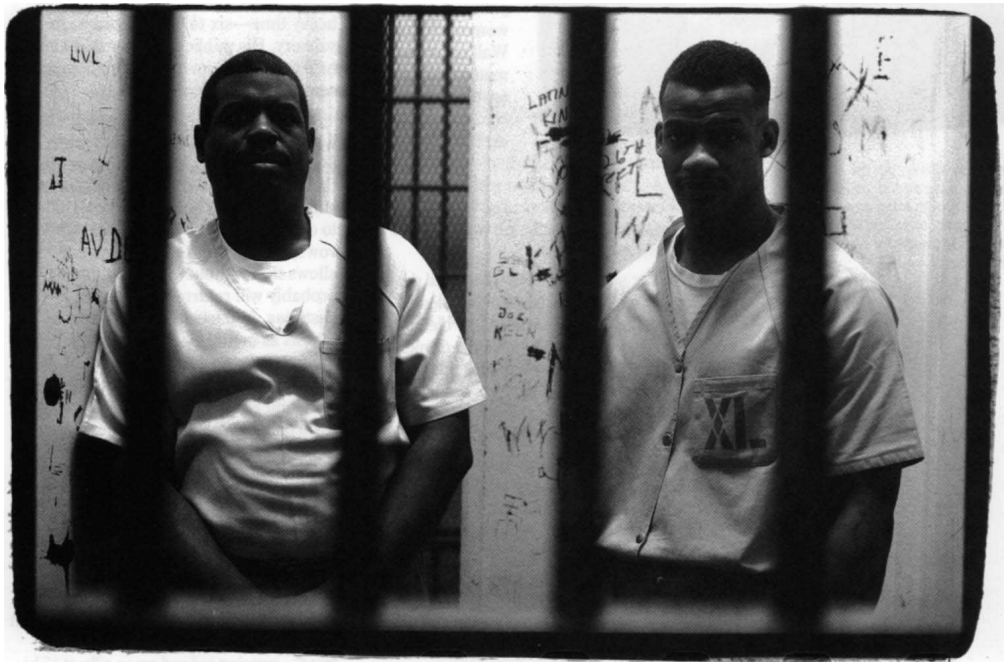

- In 2005, black males constituted around 12.5% of US men, but 389 out of every 1,000 US male inmates was black.
- Let $p = 0.125$ and $n = 1,000$: $P(k \geq 389) \approx 0$.
- 142 is maximal such that $P(k \geq 142) > 0.05$.

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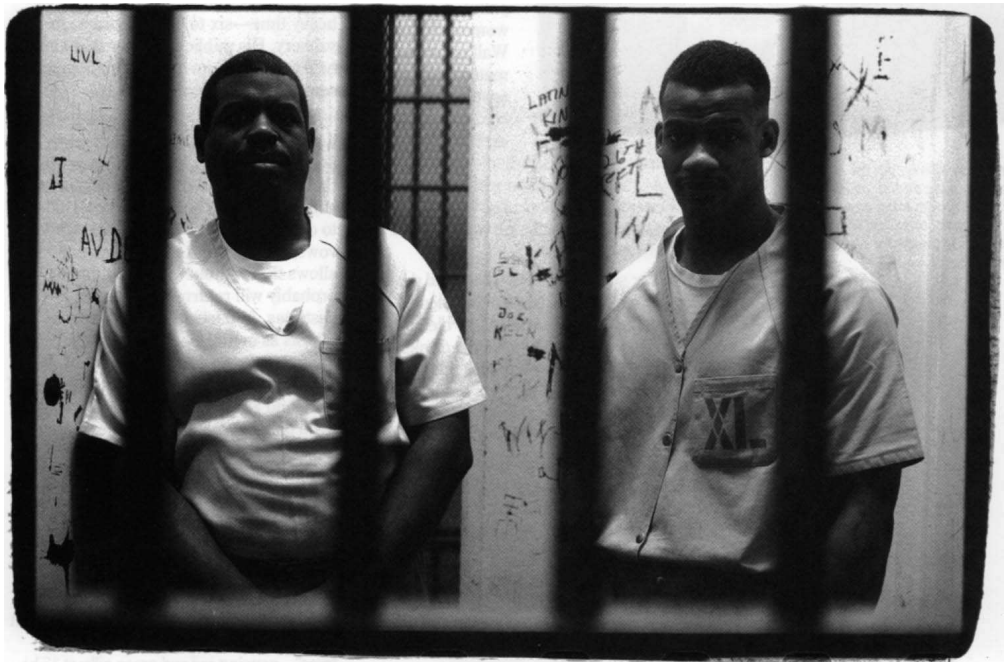

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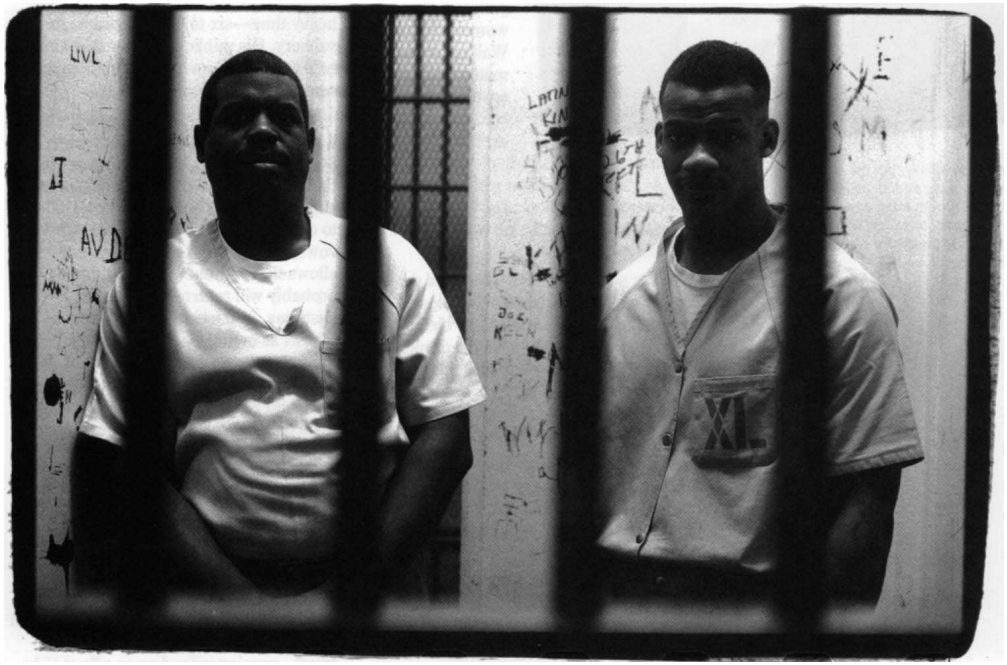

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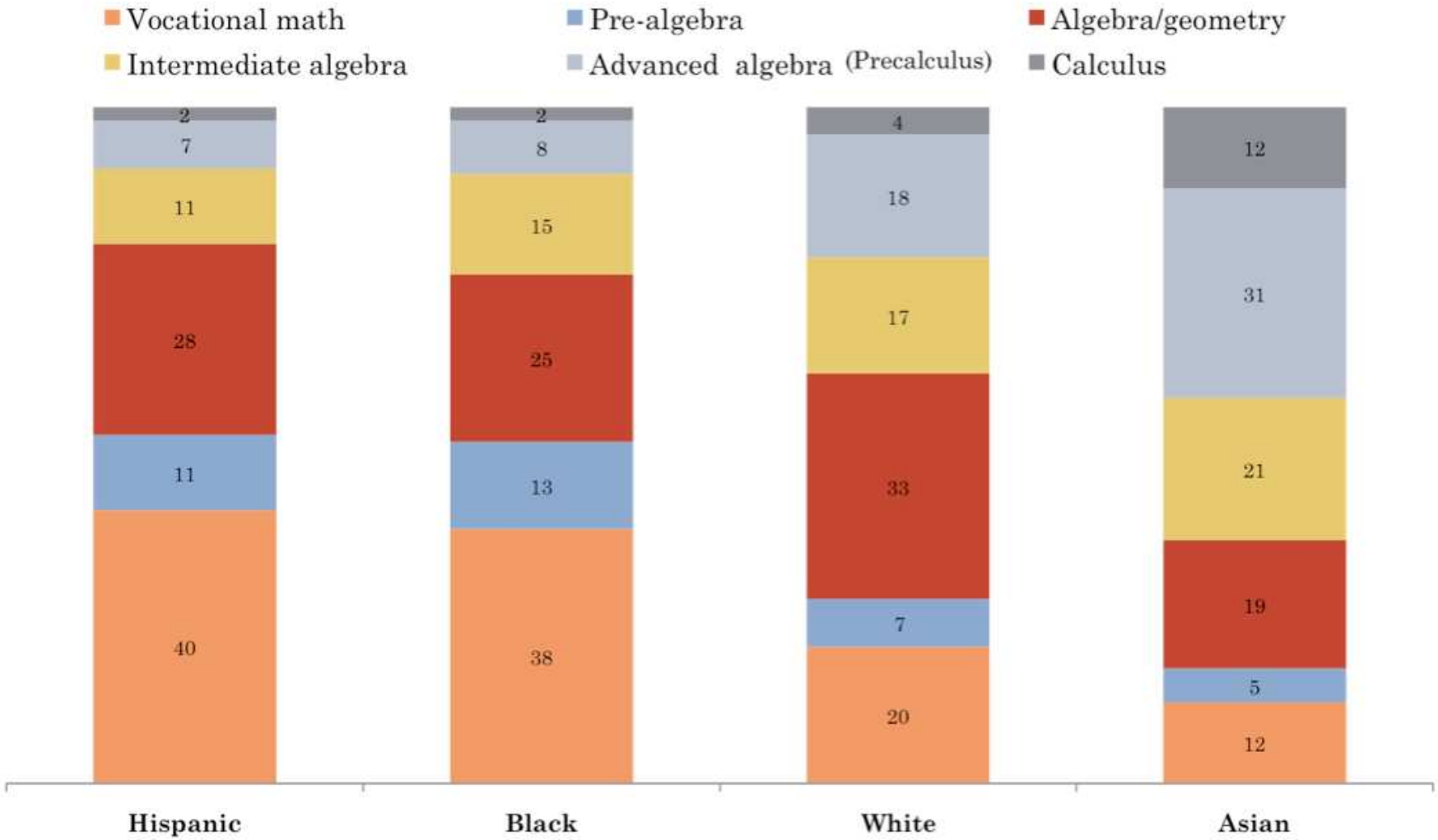
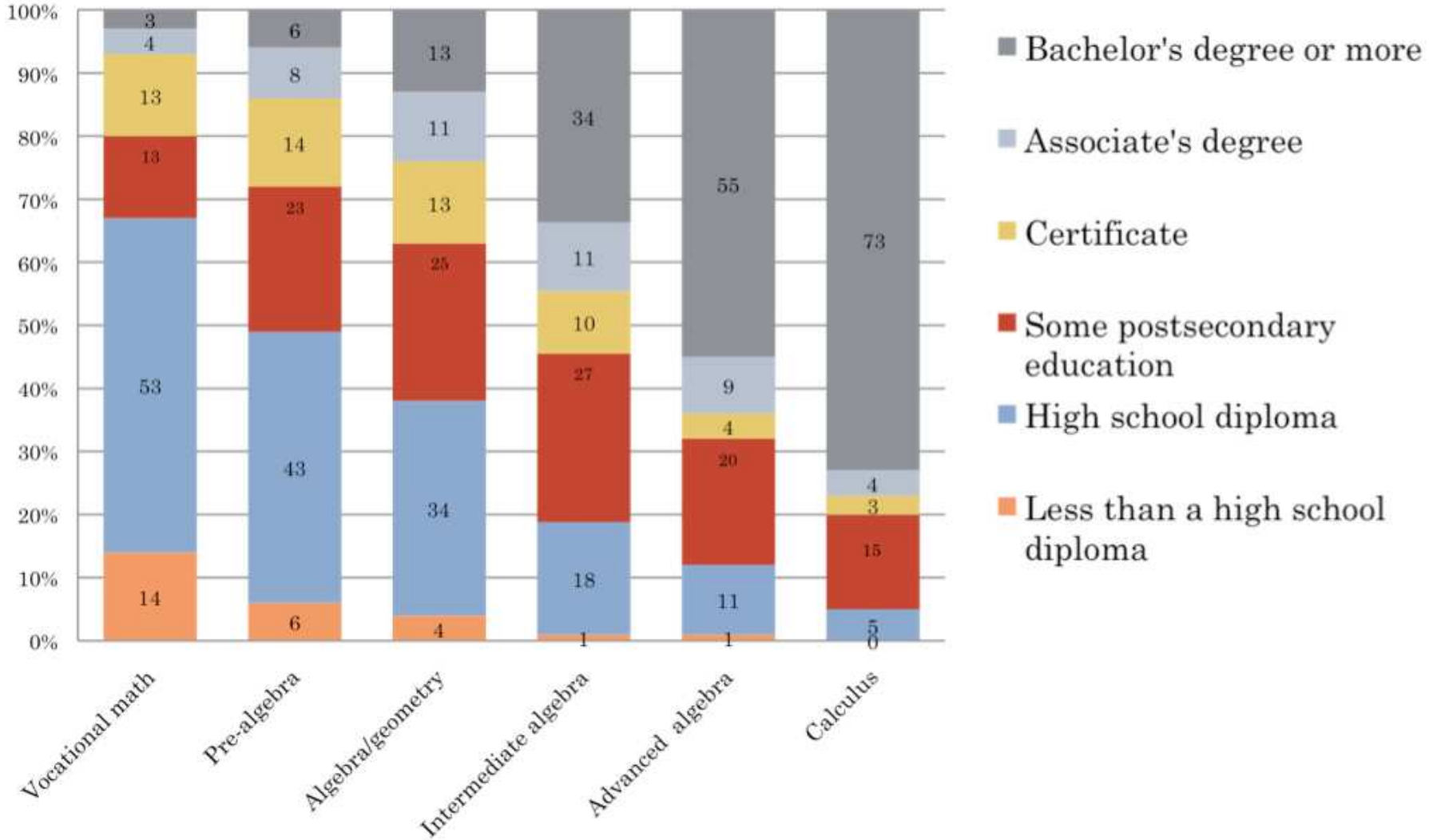
HIGHEST MATH COURSE TAKEN BY ETHNICITY



DEGREE COMPLETION



Math Achievement and Ethnicity: χ^2-Indep. Test

- What's the story at Rincon?

	Hispanic	Black	White	Asian
A	48 (58)	11 (15)	43 (40)	20 (8)
B	73 (79)	14 (21)	65 (55)	14 (11)
C	76 (96)	29 (25)	80 (66)	15 (13)
D	128 (116)	28 (31)	80 (81)	8 (16)
F	199 (175)	56 (46)	96 (122)	16 (25)

Observed vs. (Expected) Grades in Math, 2010

- Test statistic $= 52.83061161...$
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Disasters

Earthquakes: Logarithmic Functions

- 1/12/2010, Haiti: 7.0 M_W magnitude earthquake
- over 90,000 dead (some estimate 220,000)
- $M_W = \frac{2}{3}\log_{10}(E_S) - 7.5$; the energy E_S measures the destructive power.

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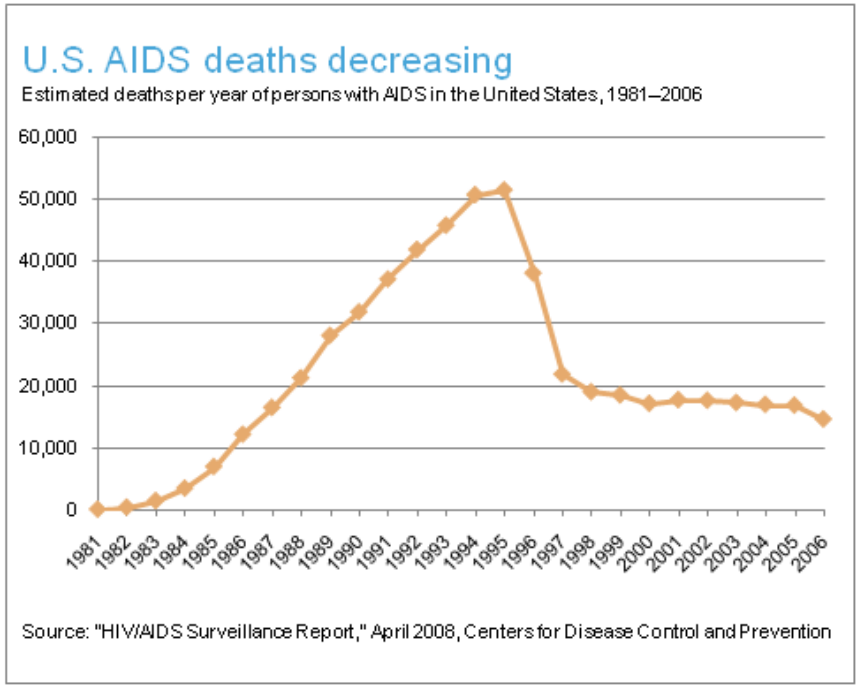

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- Piecewise model: antiretroviral drugs, awareness, sex ed

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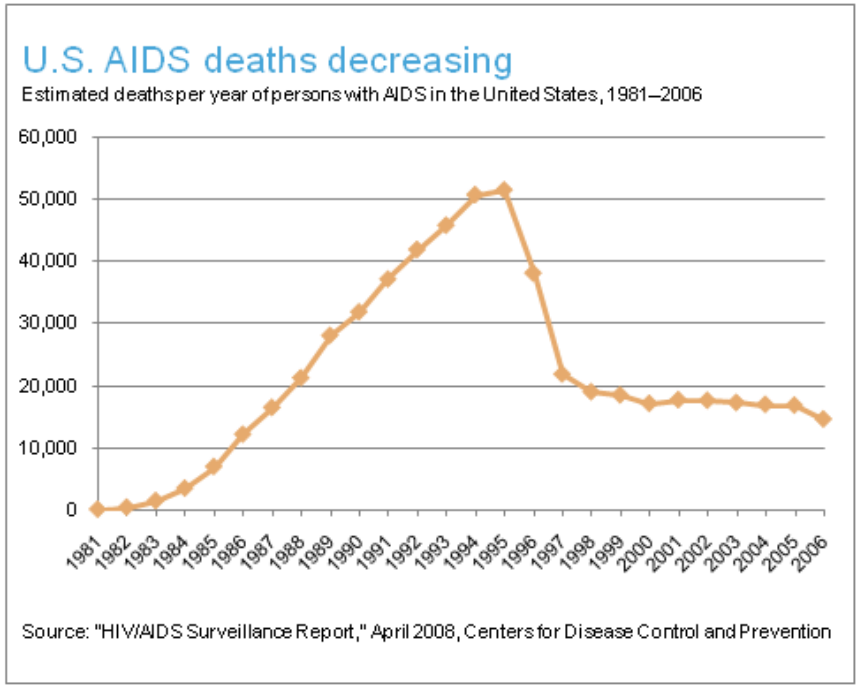

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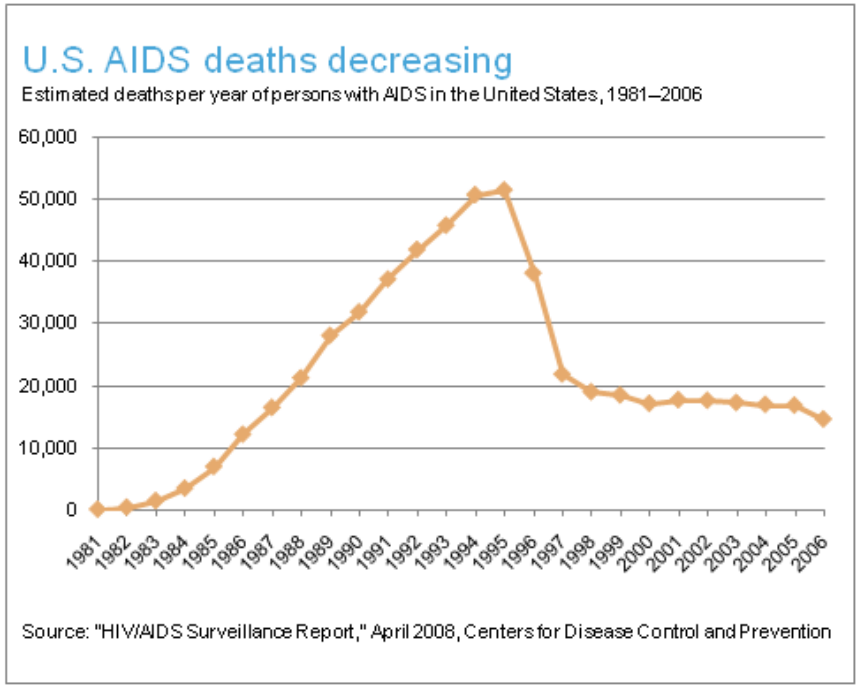

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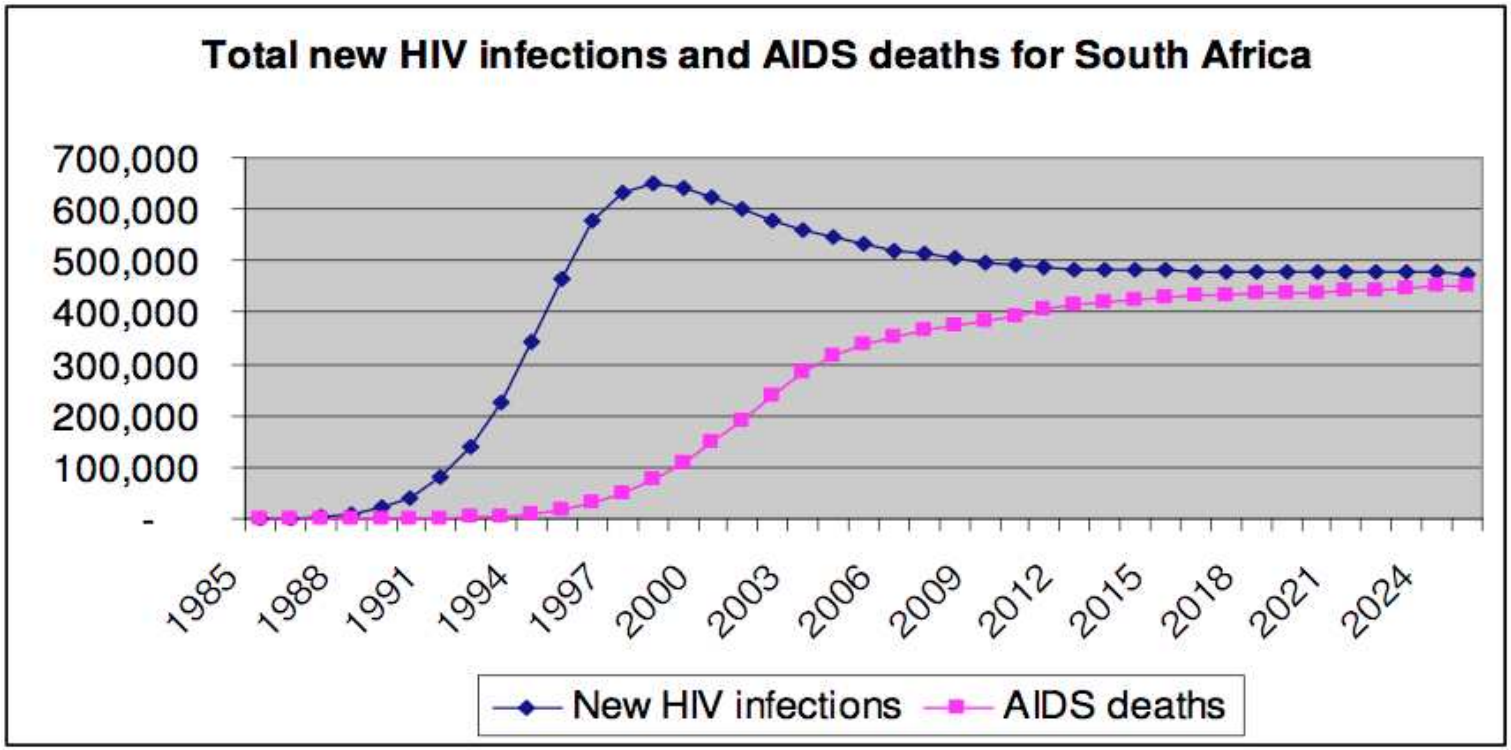

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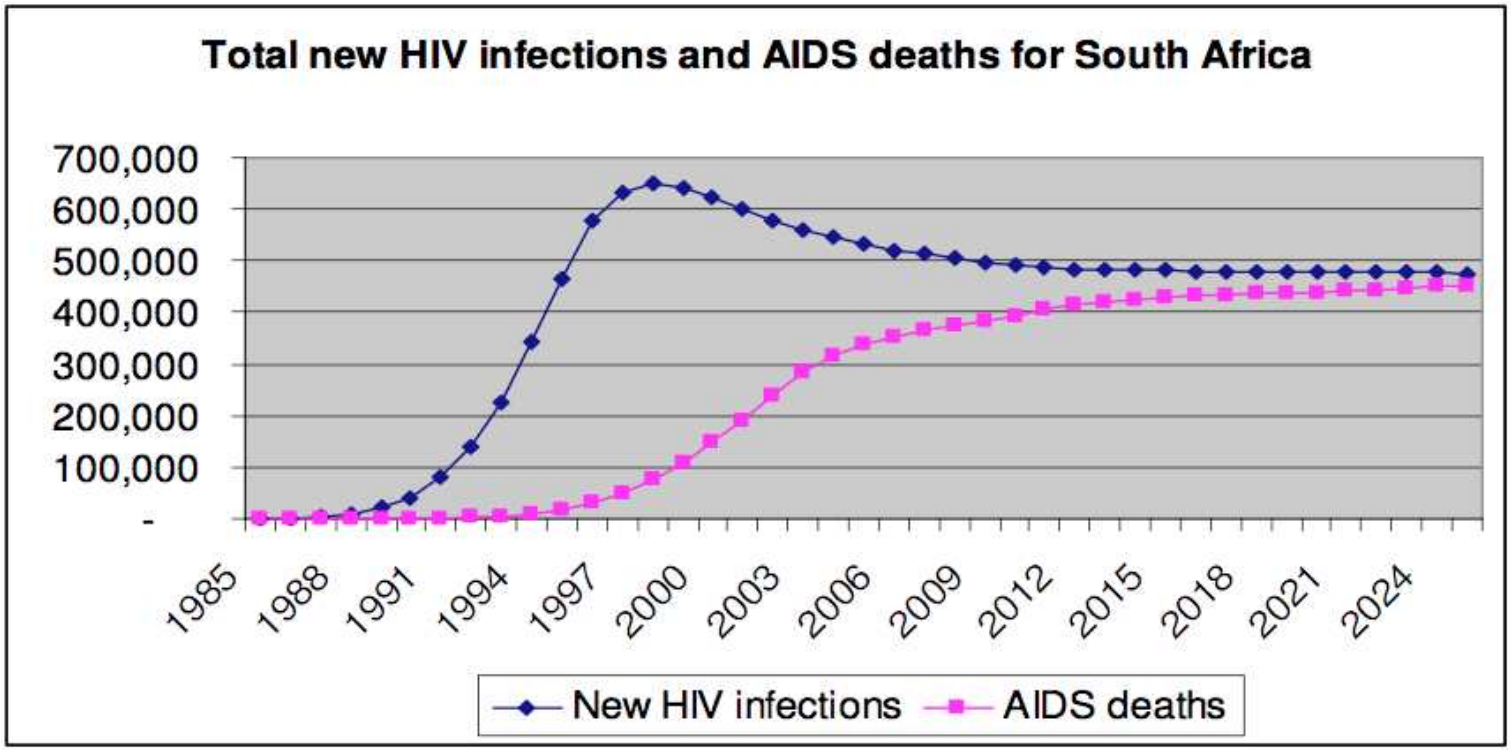

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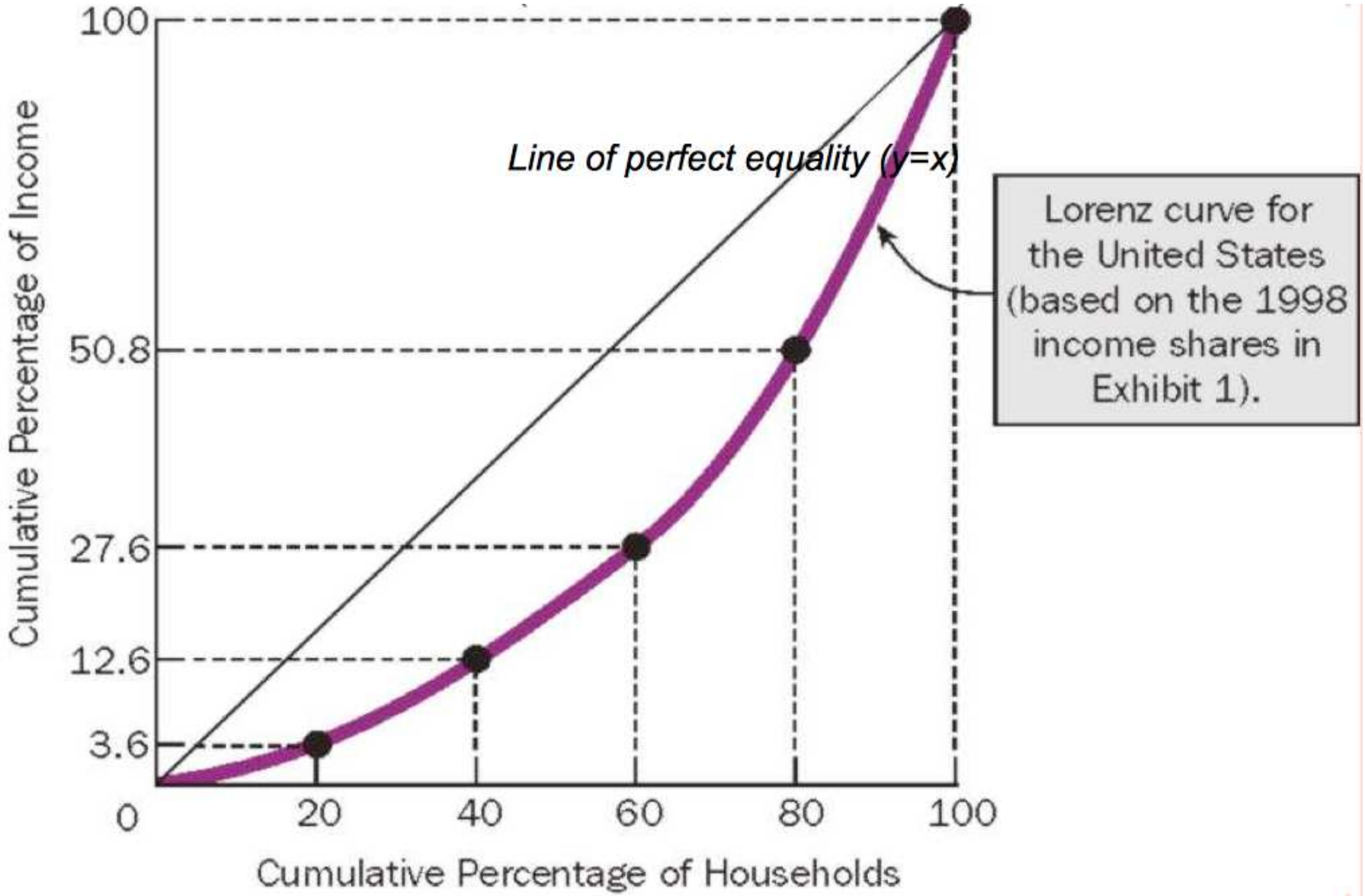
Activity: Income Inequality


100
50.8
27.6
12.6
3.6
0
20
40
60
80
100
Cumulative Percentage of Income
Cumulative Percentage of Households
Line of perfect equality (y=x)
Lorenz curve for the United States (based on the 1998 income shares in Exhibit 1).

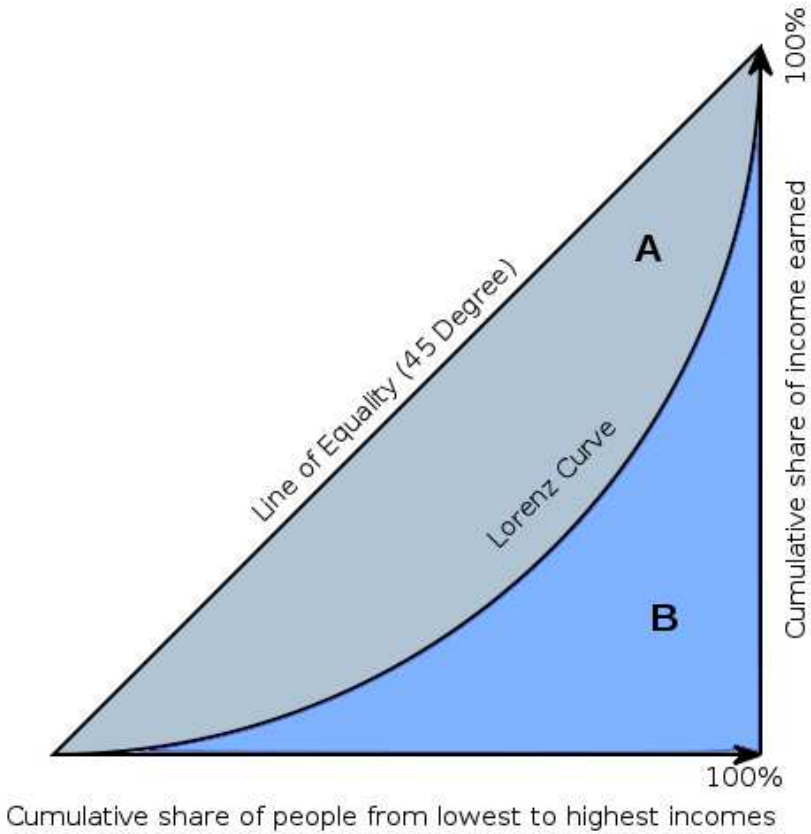
The Gini Coefficient G

- $G := \dfrac{A}{A+B}$
- If $L(x) =$ Lorenz curve,

$$B = \int_0^1 L(x)\,dx$$

- Also, $A + B = 1/2$, so

$$G = 1 - 2B$$



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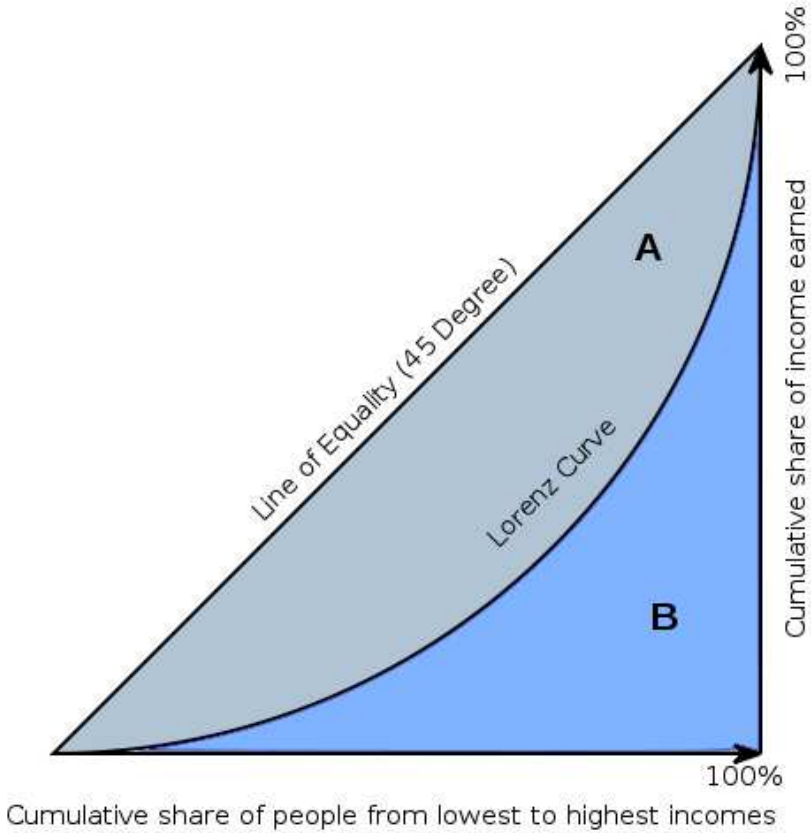
$$G = 1 - 2B$$



The Gini Coefficient G

- $G := \dfrac{A}{A+B}$
- If $L(x) =$ Lorenz curve,

$$B = \int_0^1 L(x)\,dx$$

- Also, $A + B = 1/2$, so

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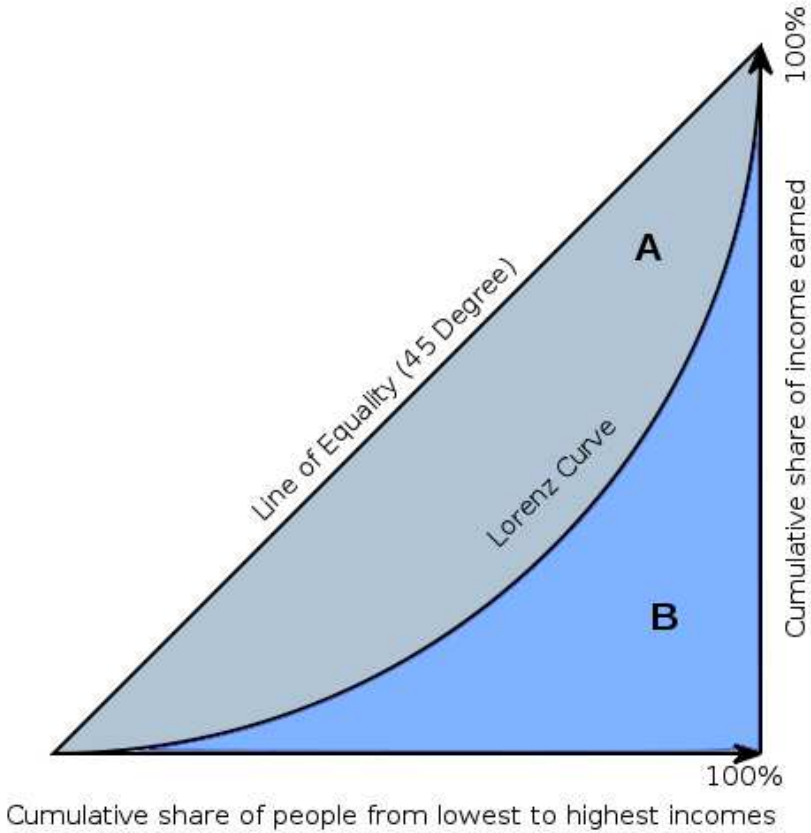
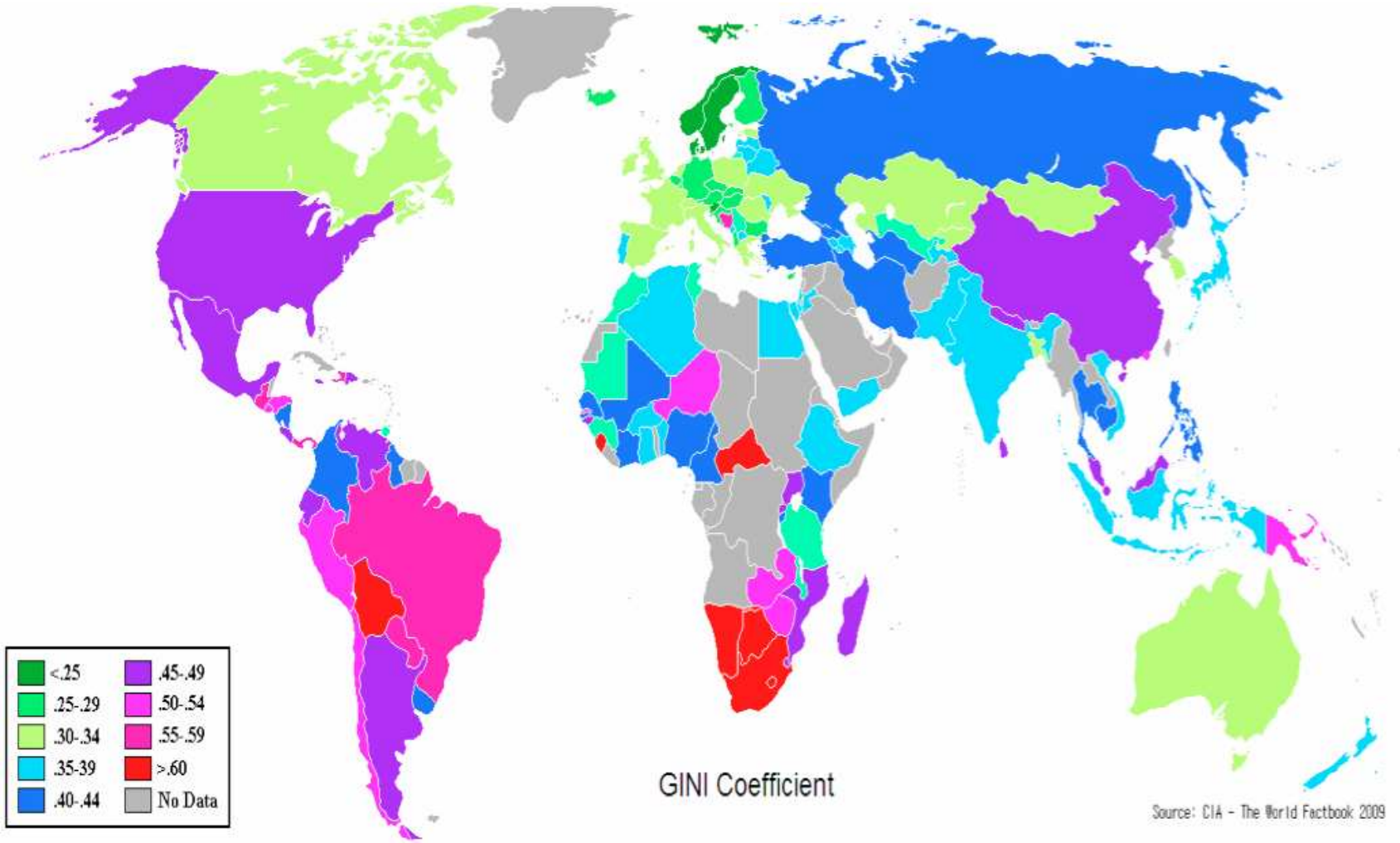
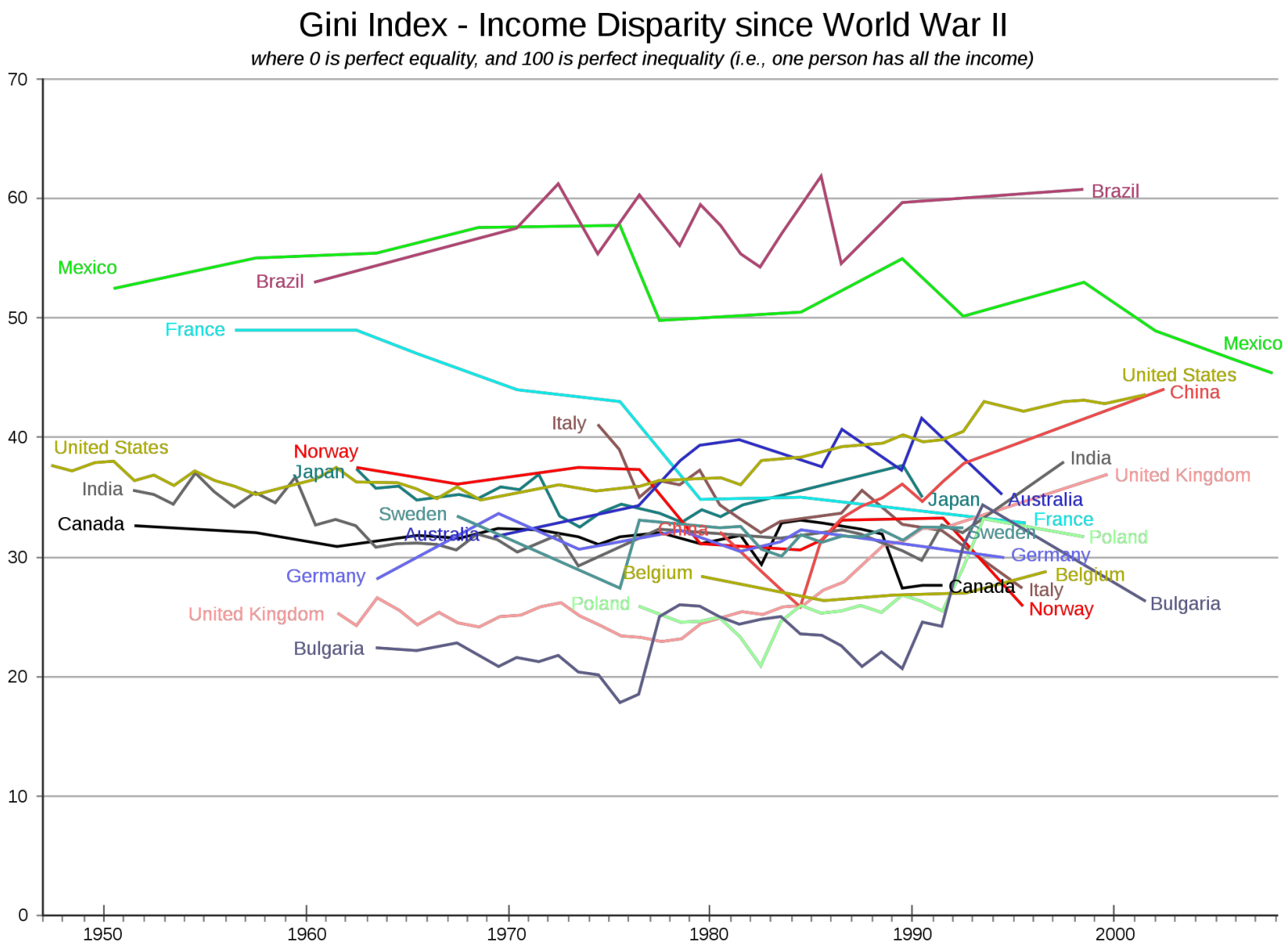


Income Inequality Worldwide




Gini Index - Income Disparity since World War II
where 0 is perfect equality, and 100 is perfect inequality (i.e., one person has all the income)
70
60
50
40
30
20
10
0
1950
1960
1970
1980
1990
2000
Mexico
Brazil
France
United States
Italy
Norway
Japan
India
Canada
Sweden
Australia
China
Germany
Belgium
Poland
United Kingdom
Bulgaria

Let's Try It!

- Enter the data: [STAT], [1]
- Find a regression equation: [STAT], [►], [7]
- Graph a regression equation: [Y=], [VARS], [5], [►], [►], [1]
- Find B: [2nd], [CALC], [7]
- Then $G = 1 - 2B$.

x	1968	1992
0.0	0.000	0.000
0.2	0.042	0.038
0.4	0.155	0.132
0.6	0.329	0.290
0.8	0.574	0.532
1.0	1.000	1.000

US Cumulative Household Income Shares by Quintile

Let's Try It!

- Enter the data: STAT, 1
- Find a regression equation: STAT, ▶, 7
- Graph a regression equation: Y=, VARS, 5, ▶, ▶, 1
- Find B: 2nd, CALC, 7
- Then $G = 1 - 2B$.

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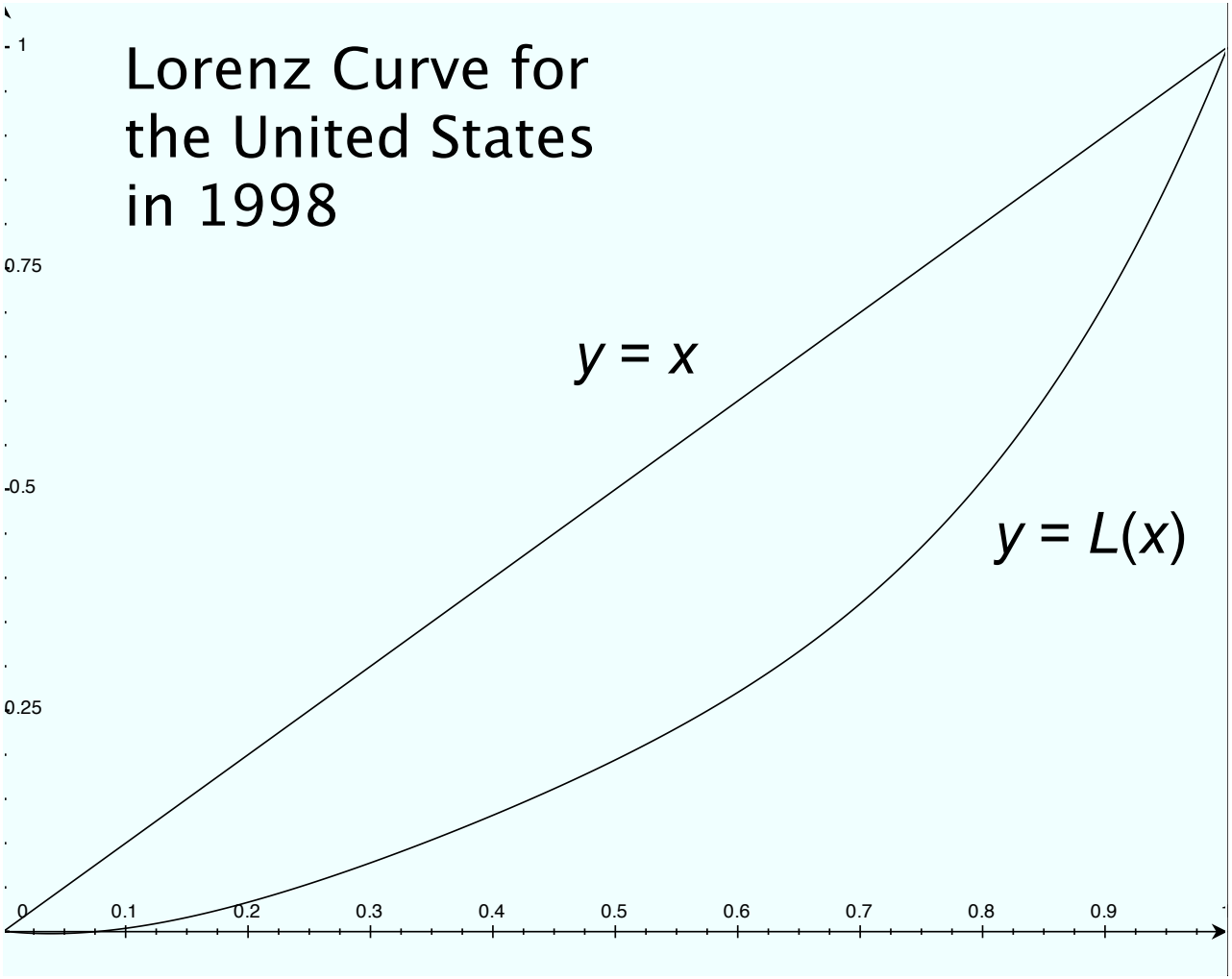
US Cumulative Household Income Shares by Quintile

Point of Maximal Inequality



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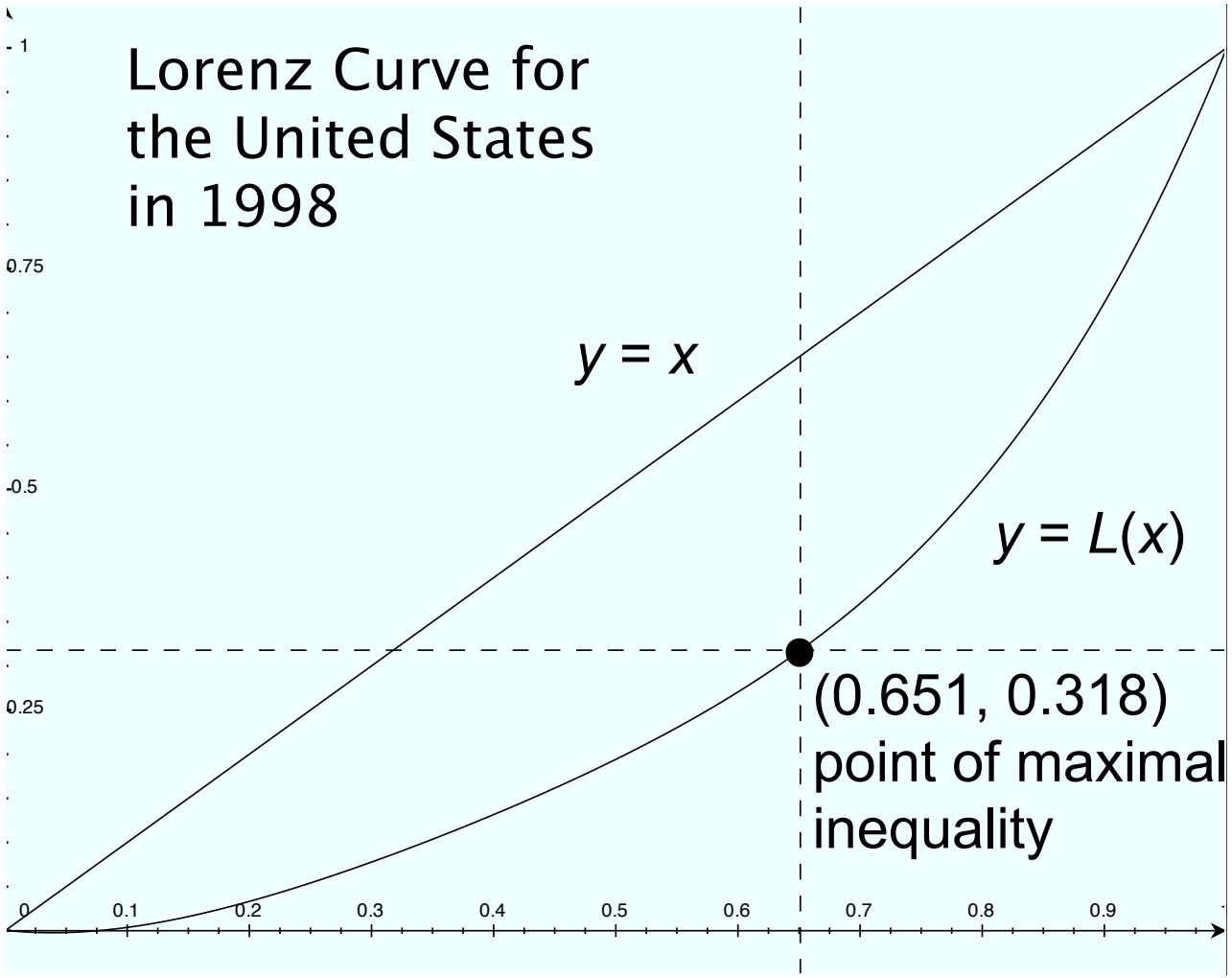

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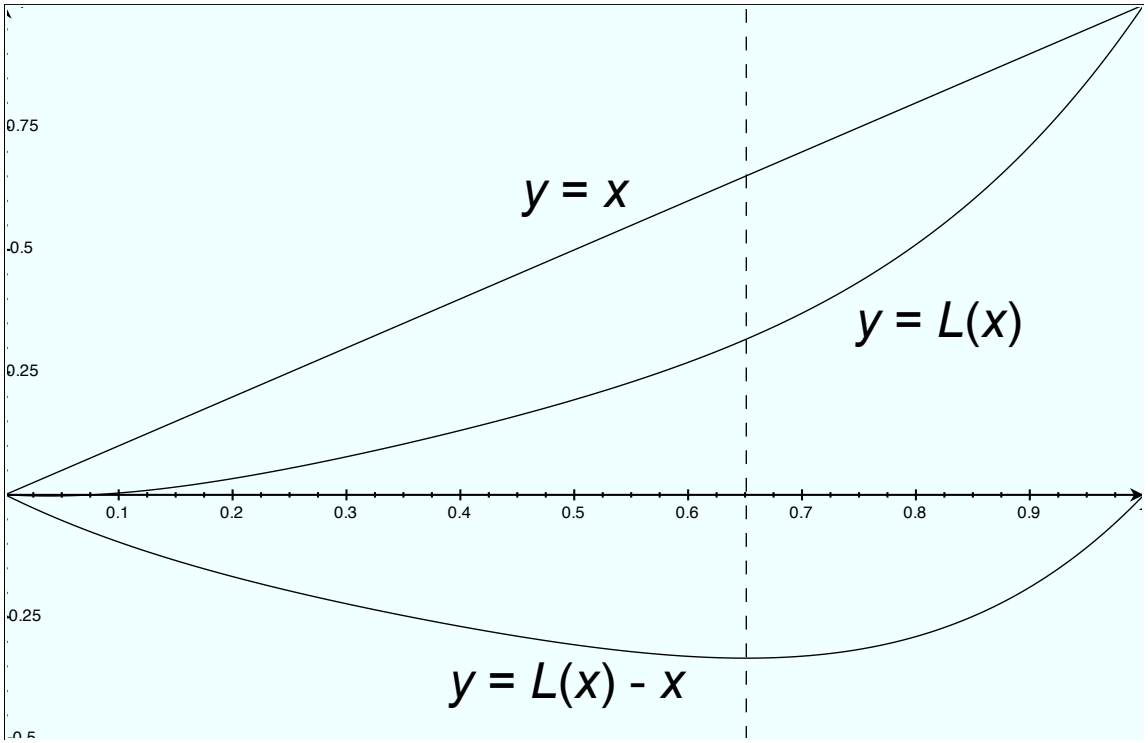

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- $L'(x) = 1$ at such a point; also, $L(x) - x$ has a minimum.

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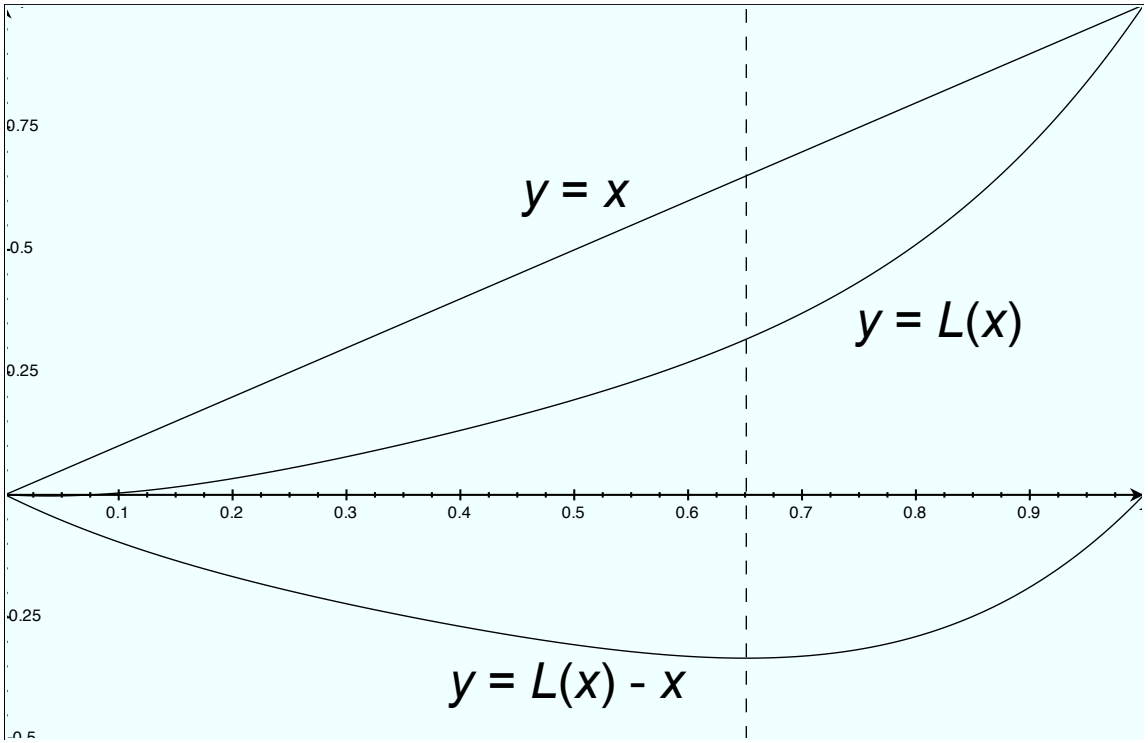

- At such a point, the tangent of $y = L(x)$ is parallel to $y = x$.
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Let's Try It!

- Subtract x from a regression equation
- Find a minimum: 2nd, CALC, 3
- Interpret the movement of this point.

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US Cumulative Household Income Shares by Quintile

Thank You!!!

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