A REVIEW OF RATIONAL TANGLES

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Abstract. The idea of a rational tangle has aided with the classification of knots. This paper reviews the general definitions of knots and tangles, combinatorial proofs of the classifications for rational tangles and poses an operation to transform knots into tangles.

1. Introduction

In the natural world knot theory has many applications, from high energy physics to jumbled sequences of DNA. Many invariants have been associated with properties of knots. From these invariants it is possible to distinguish some knots from one another, while others knots still remain unclassified. In order to determine if 2 knots are identical the crossings, for a fixed projection, are studied. Certain attributes arise from this projection, one of which is tangles. A minimal amount of knot theory knowledge is needed for understanding the material of this paper

2. Notation

For the purpose of this paper all general notation and definitions used will be defined here.

Definition 1. A ***knot*** is a closed curve embedded in a Euclidean 3-space that does not intersect itself.

Definition 2. An ***ambient isotopy*** for a knot is a continuous deformation of the knot through space. [1]

It is good to note that any knot can be embedded into a 2-sphere, which will be denoted B^3. Also, two knots are equivalent if and only if we can obtain one from the other via an ambient isotopy. From the projection representation of a knot,

Definition 3. A ***tangle*** is a proper embedding of two unoriented arcs into B^3 such that the endpoints lie on ∂B^3. Furthermore, there exist a homotopy to reduce the tangle to the trivial tangle.[2]

A 2-tangle is a planar projection of a tangle. Distinguishing between a 2-tangle and a tangle will have no effect on any computations, so in the context of this paper a tangle will refer to a 2-tangle.

[1]From the planer projection of a knot, this isotopy is formally known as a set of Reidemeister moves, see [4].

[2]For more information regarding a precise definition of tangles refer to [1]

FIGURE 1. Examples of Tangles

In order to define rational tangles, the trivial forms of tangles need to be defined. The trivial forms of tangles are two horizontal arcs or two vertical arcs. Two horizontal arcs will be denoted by $[0]$ and two vertical arcs will denoted by $\frac{1}{[0]}$ or $[\infty]$. Then $[n]$ will denote the number of horizontal half twists and $\frac{1}{[n]}$ denotes the number of vertical twists, where $n \in \mathbb{Z}$. A crossing is positive if the crossing is *right-hand oriented*for horizontal half-twists, and *left-hand oriented* for vertical half twists, otherwise the crossing is negative.

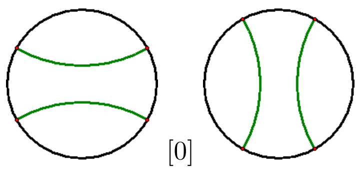


FIGURE 2. Trivial forms of Tangles

As an example, $\frac{1}{[-3]}$ describes the tangle in *Figure 1a*, additionally the tangle is rational. This fact will be made clear shortly. In order to start constructing and classifying tangles an algebraic structure needs to be defined. If T_1 and T_2 are rational tangles then two binary operations, what we will call, addition and multiplication are to be defined as follows.

$$\begin{aligned} + : (T_1, T_2) &\rightarrow T_{1+2} \\ * : (T_1, T_2) &\rightarrow T_{1*2} \end{aligned}$$

Example 1. With the addition and multiplication defined the following hold.

$$\begin{aligned} +([1],[1]) &= [2] \\ *\left(\frac{1}{[1]}, \frac{1}{[1]}\right) &= \frac{1}{[2]} \end{aligned}$$

Or in a more informal notation, which will be used for the purpose of this paper.

$$\begin{aligned} [1]+[1] &= [2] \\ \frac{1}{[1]} * \frac{1}{[1]} &= \frac{1}{[2]} \end{aligned}$$

This notation looks odd and might be unconventional, but it will be useful later.

Definition 4. A tangle is called ***simple*** if the tangle is either $[n]$ or $\frac{1}{[n]}$, $n \in \mathbb{Z}$.

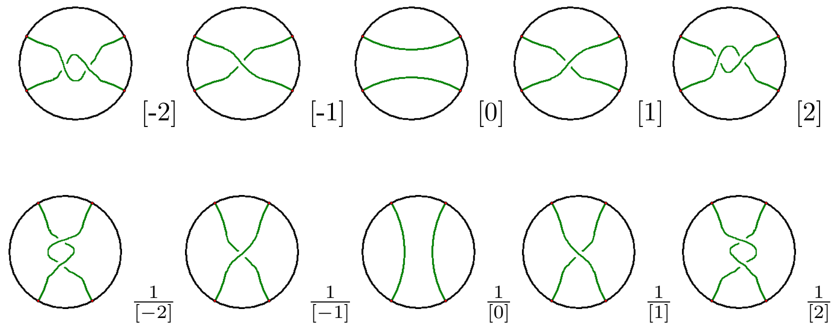
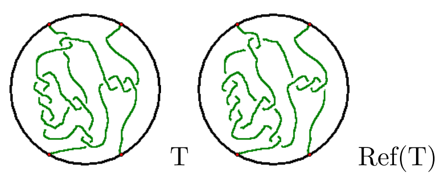


FIGURE 3. Simple Tangles

Definition 5. A tangle is called ***rational*** if the tangle can be written in the following form.

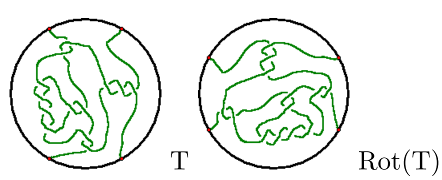
$$\sum_{i=1}^{k_1}[1] * \prod_{i=1}^{k_2}\frac{1}{[1]} + \sum_{i=1}^{k_3}[1] * \cdots + \sum_{i=1}^{k_n}[1]$$

If T is a tangle in standard form a tangle $T^{'}$ is rational if the following occur. $T^{'} = T + [n]$ or $T^{'} = T * \frac{1}{[n]}$ The last set of operations can be defined as follows. Let T and $[n]$ be tangles, then the following operations are also defined. For visual purposes, the tangle from *Figure 1b* will be used. Switching all crossings of T or $[n]$ is called a reflection.



$$Ref(T) = -T, Ref([n]) = -[n] = [-n]$$

Rotating T or $[n]$ by 90° counterclockwise.

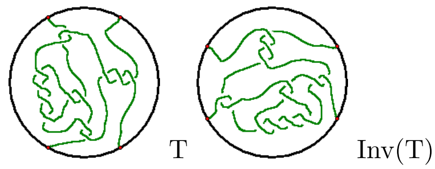


$$Rot(T) = T^r, Rot([n]) = [n]^r$$

The inverse of T or $[n]$

$$Inv(T) = -T^r, Inv([n]) = -[n]^r = [-n]^r$$

A horizontal flip is a rotation by 180° degrees about the horizontal axis and denoted by hflip(T). A vertical flip is a rotation by 180° degrees about the vertical axis and is denoted by vflip(T).



Definition 6. A ***flype*** for a tangle T is a move described by $T + [\pm 1]$ or $T * [\pm 1]$.

Before we move on the algebraic structure, it is important to note how a rational tangle is defined here. Addition and multiplication are not commutative in a normal sense. For consistency, addition will be done from the right and multiplication will be done from the bottom.

Definition 7. A tangle is in ***standard form*** if it is created by consecutive additions of simple tangles from the right and multiplications by simple tangles from the bottom.

Notice rational tangles are tangles that are ambient isotopic to a tangle in standard form. With addition, multiplication, tangle operations and some basic definitions in place an algebraic structure for rational tangles can be observed. The last definition given is numerator and denominator closure.

Definition 8. The ***numerator closure***, denoted $N(T)$, is a connecting of the poles in a horizontal manner. The ***denominator closure***, denoted $D(T)$, is a connecting of the poles in a vertical manner.

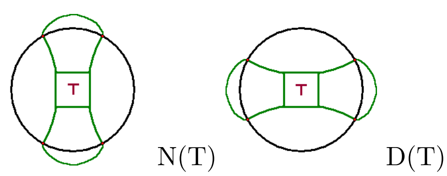


FIGURE 4. Closure of a Tangle

For the rest of this paper T will be an arbitrary rational tangle in standard form, while T_1, and T_2 will be arbitrary rational tangles.

3. Algebraic Structure

It is good to understand when a addition and/or multiplication simplify a rational tangle. Intuitively $T + (-T)$ should be either $[0]$ or $[\infty]$. Also $Inv(T) * T$ should be either $[0]$ or $[\infty]$; both of these intuitions turn out to be true. For a simple rational tangle $[n]$, it is easy to observe

$$\begin{aligned} \frac{1}{[\pm n]} + [\mp n] &= [0] \\ [\pm n] * \frac{1}{[\mp n]} &= [\infty] \end{aligned}$$

Naively following this idea tells us $Inv(T) + T = [0]$ and $T * Inv(T) = [\infty]$. In order to give a more rigorous answer to this question, observations of when and under what operations are two tangles isotopic commutative.

Lemma 1. *$hflip/vflip$ induces a rational tangle if and only T is rational.*

Proof: If $hflip(T)$ is rational then $hflip(hflip(T))$ is rational and $hflip(hflip(T)) = T$. Therefore T is rational. Conversely if T is rational then

$$\begin{aligned} T &= \sum_{i=1}^{k_1}[1] * \prod_{i=1}^{k_2}\frac{1}{[1]} + \cdots + \sum_{i=1}^{k_n}[1] \\ hflip(T) &= \sum_{i=1}^{k_n}[-1] * \prod_{i=1}^{k_2}\frac{1}{[-1]} + \cdots * \prod_{i=1}^{k_{n-1}}\frac{1}{[-1]} + \sum_{i=1}^{k_1}[-1] \end{aligned}$$

therefore $hflip$ induces a rational tangle.

Lemma 2. *T is rational if and only if $Rot(T)$ is rational.*

Proof: If T is rational in standard form, then

$$\begin{aligned} T &= \sum_{i=1}^{k_1}[1] * \prod_{i=1}^{k_2}\frac{1}{[1]} + \cdots + \sum_{i=1}^{k_n}[1] \\ T^r &= \prod_{i=1}^{k_1}\frac{1}{[-1]} + \sum_{i=1}^{k_2}[-1] * \cdots + \prod_{i=1}^{k_n}\frac{1}{[-1]} \\ &= \sum_{i=1}^{1}[0] * \prod_{i=1}^{k_1}\frac{1}{[-1]} + \sum_{i=1}^{k_2}[-1] * \cdots + \prod_{i=1}^{k_n}\frac{1}{[-1]} + \sum_{i=0}^{0}[0] \end{aligned}$$

so T^r is in standard form and therefore rational. A similar proof is done if T^r is rational.

Lemma 3. *T is rational if and only if $Inv(T)$ is rational.*

Proof: If T is rational in standard form, then

$$\begin{aligned} T &= \sum_{i=1}^{k_1}[1] * \prod_{i=1}^{k_2}\frac{1}{[1]} + \cdots + \sum_{i=1}^{k_n}[1] \\ Inv(T) = -T^r &= \prod_{i=1}^{k_1}\frac{1}{[1]} + \sum_{i=1}^{k_2}[1] * \cdots + \prod_{i=1}^{k_n}\frac{1}{[1]} \\ &= \sum_{i=1}^{1}[0] * \prod_{i=1}^{k_1}\frac{1}{[1]} + \sum_{i=1}^{k_2}[1] * \cdots + \prod_{i=1}^{k_n}\frac{1}{[1]} + \sum_{i=1}^{1}[0] \end{aligned}$$

so $Inv(T)$ is in standard form and therefore rational, and again, a similar proof is done if $Inv(T)$ is rational. Something of importance should be noted here, and that is $Inv(T) = -T^r = \frac{1}{-T}$. Immediate consequences from these lemmas are

$$\begin{aligned} T &\sim hflip(T) \\ T &\sim vflip(T) \\ T &\sim Inv(T) \\ Inv(Inv(T)) &= (T^r)^r \end{aligned}$$

Lemma 4. *Flypes are isotopic commutative, or*

$$[\pm 1] + T \sim T + [\pm 1], [\pm 1] * T \sim T * [\pm 1]$$

Proof: For $[\pm 1]+T\sim T+[\pm 1]$, Let $T^{'}=[T]+[\pm 1]$

$$\begin{aligned}
T^{'} &= \sum_{i=1}^{k_1}[1]*\prod_{i=1}^{k_2}\frac{1}{[1]}+\cdots+\sum_{i=1}^{k_n}[1]+[\pm 1]\\
&= \sum_{i=1}^{k_1}[1]*\prod_{i=1}^{k_2}\frac{1}{[1]}+\cdots+\sum_{i=1}^{k_n\pm 1}[1]\\
vflip\left(hflip(T^{'})\right) &\to \sum_{i=1}^{k_n\pm 1}[1]*\prod_{i=1}^{k_{n-1}}\frac{1}{[1]}+\cdots+\sum_{i=1}^{k_1}[1]\\
&= [\pm 1]+T^{2r}\\
Rot\left(Rot\left([\pm 1]+T^{2r}\right)\right) &= [\pm 1]+T
\end{aligned}$$

The proof for $[\pm 1]*T\sim T*[\pm 1]$ is done in a similar manner.

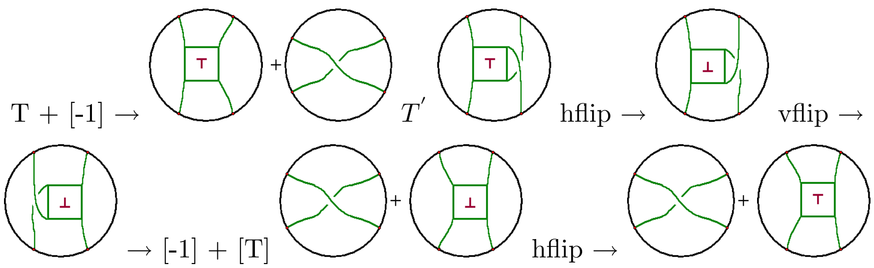


FIGURE 5. Proof of Lemma 4

Lemma 5. *Every rational tangle can be written in standard form; furthermore the standard form for a rational tangle is unique.*

Proof: By definition, a tangle is rational if it can be written in standard form. For uniqueness consider T_1 and T_2 as arbitrary rational tangles, if $T_1\sim T_2$ then let $T_1^{'}$ and $T_2^{'}$ be the standard form representation of T_1 and T_2 respectively, so $T_1\sim T_1^{'}$ and $T_2\sim T_2^{'}$. Since $T_1\sim T_2$ it is not that hard to see that $T_1^{'}\sim T_2^{'}$

$$T_1^{'}\sim T_2^{'}\Rightarrow\sum_{i=1}^{k_1}[1]*\cdots+\sum_{i=1}^{k_n}[1]\sim\sum_{i=1}^{j_1}[1]*\cdots+\sum_{i=1}^{j_m}[1]$$

Without loss of generality suppose $n>m$ and $k_n>j_m$. Counting the number of twists from the right until $T_2^{'}$ is untwisted implies

$$T_1^{'}=\sum_{i=1}^{k_1}[1]*\prod_{i=0}^{k_2}\frac{1}{[1]}+\cdots+\sum_{i=1}^{k_l}[1]\sim 0$$

So either the left over from $T_1^{'}$ is isotopic to the trivial tangle, which is a contradiction to $T_1^{'}$ being in standard form. Or $k_1,...,k_l=0$ for each k_i remaining. Which then implies that

$$T_1^{'}=\sum_{i=1}^{k_{l+1}}[1]*\cdots+\sum_{i=1}^{k_n}[1]=\sum_{i=1}^{j_1}[1]*\cdots+\sum_{i=1}^{j_m}[1]=T_2^{'}$$

or $k_{l+1}=j_1,...,k_n=j_m$. Therefore, the standard form is unique.

Lemma 6. *Addition and multiplication of rational tangles are isotopic commutative.*

Proof: If T_1 and T_2 are rational tangles, let $T_1^{'}$ and $T_2^{'}$ be their respective standard form representations. So $T_1 \sim T_1^{'}$ and $T_2 \sim T_2^{'}$. First thing that needs to be shown is that $T_1 + T_2 \sim T_1^{'} + T_2^{'}$, this is almost inherit from the definition.

$$T_1 + T_2 = T_{1+2} \sim T_{1+2}^{'} = T_1^{'} + T_2^{'}$$

So it now suffices to show $T_1^{'} + T_2^{'} \sim T_2^{'} + T_1^{'}$ since $T_1^{'} + T_2^{'} \sim T_1 + T_2$.

$$\begin{array}{rcl}
T_1^{'} + T_2^{'} & = & \sum_{i=1}^{k_1}[1] * \prod_{i=0}^{k_2}\frac{1}{[1]} + \cdots + \sum_{i=1}^{k_n}[1] + \sum_{i=1}^{j_1}[1] * \prod_{i=0}^{j_2}\frac{1}{[1]} + \cdots + \sum_{i=1}^{j_m}[1] \\
 & = & \sum_{i=1}^{k_1}[1] * \cdots + \sum_{i=1}^{k_n+j_1}[1] * \cdots + \sum_{i=1}^{j_m}[1] \\
Rot\left(Rot(T_{1+2}^{'})\right) & \rightarrow & \sum_{i=1}^{j_m}[1] * \cdots + \sum_{i=1}^{j_1+k_n}[1] * \cdots + \sum_{i=1}^{k_1}[1] \\
 & = & \sum_{i=1}^{j_m}[1] * \cdots + \sum_{i=1}^{j_1}[1] + \sum_{i=1}^{k_n}[1] * \cdots + \sum_{i=1}^{k_1}[1] \\
 & = & (T_2^{'})^{2r} + (T_1^{'})^{2r} \sim T_2^{'} + T_1^{'}
\end{array}$$

Thus $T_1 + T_2 \sim T_2 + T_1$, so addition is isotopic commutative. A similar proof is done for multiplication.

Corollary 1. *Simple tangles are isotopic commutative, or*

$$[n] + [m] \sim [m] + [n], [n] * [m] \sim [m] * [n]$$

Proof: This directly follows from *lemma 6* .

Lemma 7. *Every rational tangle satisfies the following isotopic equations*

$$T * \frac{1}{[n]} = \frac{1}{[n] + \frac{1}{T}}, \frac{1}{[n]} * T = \frac{1}{\frac{1}{T} + [n]}$$

Proof: Since multiplication is isotopic commutative $T * \frac{1}{[n]} \sim \frac{1}{[n]} * T$. So it suffices to prove $T * \frac{1}{[n]} = \frac{1}{[n]+\frac{1}{T}}$. Let $T^{'} = \frac{1}{[n]+(-T)^r}$

$$\begin{array}{rcl}
T^{'} & = & \prod_{i=1}^{n}\frac{1}{[1]} + \left[\sum_{i=1}^{k_1}[1] * \prod_{i=1}^{k_2}\frac{1}{[1]} + ... + \sum_{i=1}^{k_n}[1]\right] \\
Ref(T^{'}) & \rightarrow & \prod_{i=1}^{n}\frac{1}{[-1]} + \left[\sum_{i=1}^{k_1}[-1] * \prod_{i=1}^{k_2}\frac{1}{[-1]} + ... + \sum_{i=1}^{k_n}[-1]\right] \\
Rot(Ref(T^{'})) & \rightarrow & \left[\sum_{i=1}^{k_1}[1] * \prod_{i=1}^{k_2}\frac{1}{[1]} + ... + \sum_{i=1}^{k_n}[1]\right] * \prod_{i=1}^{n}\frac{1}{[1]} \\
 & = & T * \frac{1}{[n]}
\end{array}$$

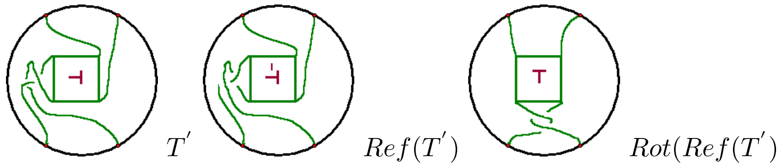


FIGURE 6. Proof of Lemma 7

4. Fractional Representations of Rational Tangles

The symetry in *lemma 7* gives an elegant way to express rational tangles. If T is in standard form then

$$T = \sum_{i=1}^{k_1}[1] * \prod_{i=1}^{k_2}\frac{1}{[1]} + ... + \sum_{i=1}^{k_n}[1]$$

where T has k_1 horizontal twists then k_2 vertical twists and so forth. For short-hand T will be represented by $[[k_1], [k_2], ..., [k_n]]$. From this we have the following proposition.

Proposition 1. *Every rational tangle can be written in a continued fraction form. If $T = [[k_1], [k_2], ..., [k_n]]$, then if $F(T)$ is the fractional representation of T.*

$$F(T) = k_1 + \cfrac{1}{k_2 + \cdots + \cfrac{1}{k_{n-1} + \cfrac{1}{k_n}}}$$

Proof: If T_1 is a rational tangle, then $T_1 \sim T$ by definition of rational tangles. Since T is in standard form, applying *lemma 7* $n-1$ times, the result is obtained. The fractional representation of T will be denoted $[k_1, k_2, ..., k_n]$. The following observations about the fractional representation of T are immediately made

$$\begin{aligned} F(T) + [\pm 1] = F(T \pm 1) &= [k_1 \pm 1, k_2, ..., k_n] \\ F\left(\frac{1}{T}\right) &= [0, k_1, k_2, ..., k_n] \\ -F(T) = F(-T) &= [-k_1, -k_2, ..., -k_n] \end{aligned}$$

Proposition 2. *Two rational tangles T_1, T_2 are equal if and only if there fractional representations are equal.*

Proof: Suppose T_1 and T_2 are in standard form, then $T_1 = [k_1, k_2, ..., k_n]$ and $T_2 = [j_1, j_2, ..., j_m]$. If $T_1 = T_2$ then standard representation forms are the same, which means that $n = m$ and $k_i = j_i$ for $i = 1$ to n. Thus their fractional representations are the same. On the other hand if T_1 and T_2 have the same fractional representations then $T_1 = [k_1, k_2, ..., k_n] = T_2$, thus $T_1 = T_2$.

The next few proofs rely on some knowledge between and continued fractions and matrix theory.

Proposition 3. *Let $N(T)$ denote the numerator closure of a rational tangle T. Then if $N(\frac{p}{q})$ is a knot, $N(\frac{p}{q})$ is p-colorable.*

Proof: Let $F(T) = [k_1, ..., k_n]$, then let the matrix representation of F(T) be denoted by

$$\begin{aligned} M(T) &= \begin{pmatrix} k_1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} k_2 & 1 \\ 1 & 0 \end{pmatrix} \cdots \begin{pmatrix} k_n & 1 \\ 1 & 0 \end{pmatrix} \\ &\rightarrow \quad M(T) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} \\ &\rightarrow \quad \frac{p}{q} = \frac{Det([\nabla_{N(T)}(-1)])}{Det([\nabla_{D(T)}(-1)])} \end{aligned}$$

Where $Det([\nabla_{N(T)}(\chi)])$ is a representation for the Alexander-Conway polynomial for $N(T)$. Which directly implies that $Det([\nabla_{N(T)}(-1)]) = p$.[3]

Corollary 2. *If $F(T_1) = \frac{p}{q}$ and $F(T_2) = \frac{r}{s}$, $p = r$ and $qs^{\pm} \equiv 1$ mod p, then there exist a finite number of flypes such that $T_1 \sim T_2 \pm [n]$, where $n \in \mathbb{N}$.*

Proof: Let $q = np \pm s$, all that needs to be shown is $q = p \pm s$, then $q = np \pm s$ follows by induction. If $q = p \pm s$ then $r = q \mp s$ so

$$\frac{p}{q} = \frac{q \mp s}{s} = \frac{q}{s} \mp 1$$

Let $F(T_3) = \frac{q}{s}$, then $F(T_3) = [j_1, ..., j_m]$ so $T_3 = [[j_1], ..., [j_m]]$. This implies that

$$\begin{aligned} T_2 = T_3 \mp [1] &= 1 \mp [[j_1], ..., [j_m]] \\ &= [[j_1 \mp 1], ..., [j_m]] \\ &= [[k_1], ..., [k_n]] = T_1 \end{aligned}$$

Where $[[k_1], ..., [k_n]] = \frac{p}{q}$. Thus $T_1 \sim T_2 \pm [1]$, then by induction the result follows.

Another consequence that can be applied to $N(T)$ is the following. For those familiar with knot theory, this is a simplified statement of Conway's theorem.

Proposition 4. *$N(\frac{p}{q}) = N(\frac{r}{s})$ if and only if $p = r$ and $qs^{\pm 1} \equiv 1$ mod p*

Proof: Let $F(T_1) = \frac{p}{q}$ and $F(T_2) = \frac{r}{s}$, if $p = r$ and $qs^{\pm} \equiv 1$ *mod* p, then by the corollary of *Proposition 3*, $T_1 \sim T_2$. So then $N(T_1) \sim N(T_2)$, but if $N(T_1) \sim N(T_2)$, then both knots are isotopic, which implies there exist a finite set of Reidemeister moves to transform $N(T_1)$ to $N(T_2)$, therefore $N(T_1) = N(T_2)$. On the other hand, if $N(\frac{p}{q}) = N(\frac{r}{a})$ this imples that $T_1 \sim T_2$. Since

$$\frac{p}{q} = \frac{Det([\nabla_{N(T)}(-1)])}{Det([\nabla_{D(T)}(-1)])} = \frac{r}{s}$$

$p = Det([\nabla_{N(T)}(-1)]) = r$ which means that T is r and p colorable, which implies $p|r$ and $r|p$ hence $p = r$. Also, by the corollary of *Proposition 3*, if $N(T_1) = N(T_1)$, then $T_1 \sim T_2 \pm [n]$. So if $q = np \pm s$ then $T_1 \sim T_2 \pm [n]$ by the same proof. The last case is when $q = np \pm s \pm a$, where $a \in [1, p-1]$. A simple example shows this to be false. Let $F(T_1) = \frac{5}{7}$, $F(T_2) = \frac{5}{3}$, $a = 1$, so $21 \equiv 1$ *mod* 5

$$\frac{5}{7} = \frac{\frac{7+3}{2}}{7} \neq \frac{\frac{7+3+1}{2}}{7} = \frac{11}{14}$$

So $N(T_1)$ and $N(T_2)$ are 5-colorable but $N(\frac{11}{14})$ is 11-colorable. Thus $q = np \pm s$.

[3]The matrix product $M(T)$ is a representation of the Euclidean algorithm of $\frac{p}{q}$.

5. Knot Actions

This section is solely for future research purpose. A crossing on a planar projection of a knot can be given a polarity. Depending on orientation, a crossing can be either $+1$ or -1. For a planar projection of a knot, breaking an arbitrary crossing is the same as breaking the knot in two locations in 3-space. The next definition is for a fixed projection.

Definition 9. A ***knot action*** κ, for a projection of a knot with n crossings, is a breaking of a crossing where the two ends of the overcrossing are give a positive polarity, while the two ends of the undercrossing are given a negative polarity. Furthermore, the knot can be represented by a tangle with $n-1$ crossings.

A couple simple examples can show that $N(T)$ and $D(T)$ might not necessarily represent a knot/(link) action. Below is an example of a knot action on the knot 8_{10} from Rolfsens knot table.[4]

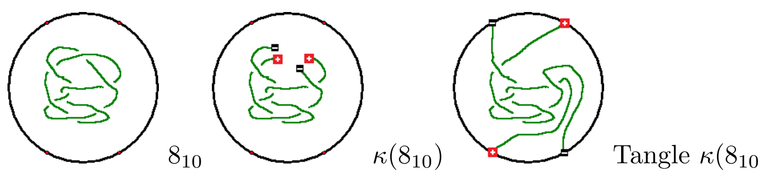


Figure 7. Example of a Knot Action on 8_{10}

To reiterate, in 3-space a knot action breaks a closed curve into 2 unoriented arcs. The above definition also applies to a link. For every finite knot, there exist a finite number of knot actions up to isotopy. This leads to the following claim.

Claim 1. *Every tangle represents a knot or link action*

Proof: Given a tangle, assign the two top poles a positive and negative polarity, and the bottom poles a positive and negative polarity such that $N(T)$ and $D(T)$ connect opposite polarities.

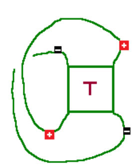


Figure 8. A tangle representing a knot action

Claim 2. *Knot/(link) action are invariant under the Reidemeister moves.*

[4]Rolfsens knot table can be found in many books and online. This paper references[5], for the table

Proof: If K is a knot with n crossings then $\kappa(K) = T$ with $n-1$ crossings. Tangles are invariant under the Reidemeister moves, thus $\kappa(K)$ is invariant under Reidemeister moves. Since $\kappa(K)$ inherits the algebraic structure for tangles, then it follows that addition of rows and multiplication of columns are invariant under the Reidemeister moves.

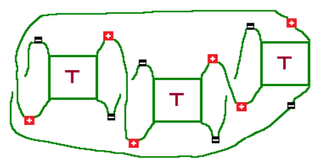


FIGURE 9. A more complicated structure of a Knot Action

Switching the polarity of a knot action changes the original knot, for now this will be referred to as a polarity shift. It should be clear that a set of knot actions and polarity shifts are the operations that produce the unknotting number for a knot. In order to start analyzing this idea, knowledge of homology theory and manifolds is needed, which is beyond the purpose and scope of this paper.

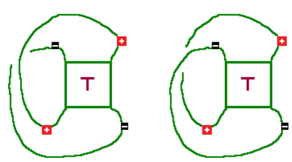


FIGURE 10. Polarity Shift of a Knot Action

6. Conclusion

Tangles form a minor section of knot theory. From basic observations of a projection, simple ideas of reflecting, rotating and twisting turn into a rather sophisticated algebraic structure. This structure turns out to be a way of distinguishing two projections, not only of tangles but knots as well. From this structure and projections some knots can be distinguished from one another in 3-space as well. All of these ideas build up to much more complicated and complete invariants for knots. Most of which appear in topological concepts relating much of our physical world. In summary, rudimentary ideas can lead to the most complex of things.

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