

Name:

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Math 2B: Quiz 2

You **DO NOT** need to simplify any algebraic expressions.

(5) **1.** Use the trapezoidal rule with $n = 4$ to approximate the following integral. $\int_1^3 \frac{1}{x} dx$

$\Delta x = \frac{1}{2} \Rightarrow$ the partition endpoints for $[1, 3]$ are $\{1, \frac{3}{2}, 2, \frac{5}{2}, 3\}$. By the trapezoidal rule we have

$$\int_1^3 \frac{1}{x} dx \approx \frac{\Delta x}{2} \left[f(1) + 2f\left(\frac{3}{2}\right) + 2f(2) + 2f\left(\frac{5}{2}\right) + f(3) \right] = \frac{1}{4} \left[1 + \frac{4}{3} + 1 + \frac{4}{5} + \frac{1}{3} \right] = \frac{67}{60} \quad \square$$

(5) **2.** Find a bound for the error of the approximation in problem 1.

$$\text{Hint: } |ET_n| \leq K \frac{(b-a)^3}{12n^2}$$

$$|f''| \leq K, \text{ so } f'(x) = \frac{-1}{x^2}, f''(x) = \frac{2}{x^3} \Rightarrow K = \max_{x \in [1, 3]} f''(x) = 2 \text{ then } |ET_n| \leq 2 \frac{(2)^3}{12(4)^2} = \frac{1}{12}$$

So the error is bounded by $\frac{1}{12} \quad \square$