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Math 2B: Quiz 8

(5) **1.** Integrate the following using partial fractions $\int \frac{x^3+x}{x^2-1}$

After polynomial division we have

$$\int \frac{x^3+x}{x^2-1}dx = \int x + \frac{2x}{x^2-1}dx$$

Then using partial fractions on the second term

$$\frac{2x}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} \qquad \Rightarrow \qquad 2x = A(x+1) + B(x-1)$$

If $x = 1$, then we have $2 = 2A$, so $A = 1$
If $x = -1$, then we have $-2 = -2B$, so $B = 1$, hence our decomposition is

$$\frac{1}{x-1} + \frac{1}{x+1}$$

which implies our integral

$$\begin{aligned} \int \frac{x^3+x}{x^2-1}dx &= \int x + \frac{1}{x-1} + \frac{1}{x+1}dx \\ &= \frac{x^2}{2} + \ln|x-1| + \ln|x+1| = C \end{aligned}$$

(5) **2.** Determine whether the integral converges or diverges, if it converges find the limit. $\int_0^1 \frac{x}{x^2-1}$

$$\begin{aligned} \int_0^1 \frac{x}{x^2-1} &= \lim_{t\to 1^-} \int_0^t \frac{x}{x^2-1} \\ &= \lim_{t\to 1^-} \frac{1}{2}\ln|x^2-1|\Big|_0^t \\ &= \lim_{t\to 1^-} \frac{1}{2}\ln|t^2-1| \\ &= -\infty \end{aligned}$$

$\therefore$ the integral is divergent.