Additional Problems

1. Use the definition of the definite integral to evaluate $\int_1^3 4x^3 - 3x^2 dx$

2. Find the average value of the function $f(x) = -x^2 + 1$ on $[-1, 1]$, and where does the average value occur.

3. Find $f'(x)$, when $f(x) = \int_{x^3-1}^{\tan(x)} y^3 - 5dy$

4. Find the equation of the tangent line of $f(x) = \int_0^x \sin\sqrt{t^2 + \pi^2} dt$ when $x = 0$

5. evaluate the following integrals

(a) $\int \frac{\sin(x)}{\sqrt{\cos(x)}} dx$

(b) $\int \frac{2x}{(x^2+8)^2} dx$

(c) $\int \frac{\sqrt{1+\sqrt{x}}}{2\sqrt{x}} dx$

(d) $\int_0^{\frac{\pi}{4}} \sec(x)\tan(x) dx$

(e) $\int_0^{\frac{\pi}{2}} \sin(4x)\cos(3x) dx$ *HINT: angle sum formulas*

6. Approximate the following with the midpoint rule, trapezoidal rule with $n = 5$ and find a bound for the error of each approximation. $\int_0^1 4\sqrt{1-x^2}$

7. For problem 7 find the number of partitions need to guarantee an accuracy for the following (a) $|EM_n| = 10^{-5}$ (b) $|ET_n| = 10^{-7}$

8. Find area between the following curves

(a) $f(x) = \cos(x), g(x) = \sin(x)$ on $[0, \pi]$

(b) $y = x, y = -x, x = 1$

9. Find the volume of the solid formed by revolving the given region ($\mathcal{R}$) about the line $y = 1$, $\mathcal{R} = \{f(x) = 2x, g(x) = 2, x = 0\}$

10. Find the volume of the solid formed by revolving the given region ($\mathcal{R}$) about the line $x = 3$, $\mathcal{R} = \{y = x, y = x^2 - 2\}$