Problem #1: Line and Surface Integrals. Do the following:
(a) Let \( f(x, y, z) = 3x + xy + z^3 \) and let \( c(t) = (\cos(4t), \sin(4t), 3t), t \in [0, 2\pi] \). Find \( \int_C f \, ds \).
(b) Let \( c(t) \) be a path and \( T \) the unit tangent vector. What is \( \int_C T \cdot ds \)?
(c) Let \( c(t) = (e^{2t}\cos(3t), e^{2t}\sin(3t)), t \in [0, 2\pi] \). Find \( \int_C \frac{x \, dx + y \, dy}{(x^2 + y^2)^{3/2}} \).
(d) Set up but do not evaluate an integral that represents the surface area of the ellipsoid \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \).
(e) Let \( D \) be the dome shaped region bounded by \( z = 8 - 2x^2 - 2y^2 \) and the \( xy \)-plane and let \( S \) be the boundary surface of \( D \). Let \( f(x, y, z) = x^2 + y^2 + 3(z-2)^2 \) and let \( F = \nabla f \). Calculate the flux of \( F \) through \( S \).

Problem #2: Use Green’s Theorem. Do the following:
(a) Evaluate \( \oint_C (x^2 - y^2) \, dx + (x^2 + y^2) \, dy \) where \( C \) is perimeter of the rectangle with vertices \((0, 0), (2, 0), (0, 1), \) and \((2, 1)\).
(b) Show that for any closed curve \( C \) in the plane \( \oint_C 3x^2y \, dx + x^3 \, dy = 0 \).
(c) Sketch the curve given parametrically by \( c(t) = (1 - t^2, t^3 - t) \) and find the area enclosed by it.

Problem #3: Use Divergence or Stokes’ Theorem. Do the following:
(a) Let \( S \) be the surface defined by \( x^2 + y^2 + 5z = 1, z \geq 0 \). and let \( F(x, y, z) = (xz, yz, x^2 + y^2) \). Verify Stokes’s theorem for this surface and vector field.
(b) Let \( S \) be the surface defined by \( y = 10 - x^2 - z^2, y \geq 1 \) and let \( F(x, y, z) = (2xyz + 5z, e^x \cos(yz), x^2y) \). Find \( \iint_S \nabla \times F \cdot d\mathbf{S} \).
(c) Let \( S \) be the surface defined by \( z = e^{1-x^2-y^2}, z \geq 1 \) and let \( F(x, y, z) = (x, y, 2z-2z) \). Calculate \( \iint_S F \cdot d\mathbf{S} \).