NOTES FOR OCTOBER 8, 2004

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It is ingrained in mathematical science that every real advance goes hand in hand with the invention of sharper tools and simpler methods which at the same time assist in understanding earlier theories and cast aside older more complicated developments.

David Hilbert (1862-1943)

1. Some closely related concepts

In class, I mentioned the concepts of a "lattice of sets", a "σ-algebra" and a "complete lattice of sets". Much more time was spent talking about the basic idea of a category with examples (sets and functions, groups and group homomorphisms, metric spaces and continuous maps, topological spaces and continuous maps) and functors between categories. The following two propositions establish that topological spaces and continuous maps for a category

Proposition 1.1. *For any topological space A, the identity map $f : A \to A$ is continuous.*

Proposition 1.2. *If $f : A \to B$ and $g : B \to C$ are continous maps between topological spaces, then $f \circ g$ is continous.*

The next proposition shows that the map from the category of metric spaces to the category of topological spaces is a functor.

Proposition 1.3. *If $f : A \to B$ is a map between metric spaces, then f is continuous (in the metric sense), if and only if it is continuous (in the topological sense) as a map between the associated topological spaces.*

The transition from metric spaces to topological spaces can be thought of as "learning to reason without a metric". The transition to cateogory theory can be thought of as "learning to reason without elements".

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