Coxeter groups, Artin groups, and non-positive curvature

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Abstract. These lecture notes are derived from a special topics course taught at the University of California, Santa Barbara during spring quarter, 2004. The notes are divided into three parts. The first chapter introduces Coxeter groups and their basic properties and the second focuses on the theory of non-positive curvature, specifically the theory of CAT(0) spaces and groups. The major goal of the first two chapters is a full proof of Moussong's theorem that all Coxeter groups are CAT(0) groups. In the final chapter, Artin groups and Garside structures are introduced and their curvature properties are surveyed.

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