Math 110, Fall 2012, Sections 109-110
Worksheet 10

1. True or false:

   (a) If A and B are row equivalent, then A is diagonalizable if and only if B is diagonalizable.

   (b) If A is diagonalizable, then $\dim E_\lambda$ is equal to the multiplicity of λ for all eigenvalues λ.

   (c) The multiplicities of all eigenvalues of a matrix add up to the size of the matrix

2. Are the following matrices diagonalizable? Try to answer without taking a single determinant or doing a single row operation.

$$A = \begin{pmatrix} -99 & 42 & 16 \\ 0 & e & -12 \\ 0 & 0 & 432 \end{pmatrix}, \qquad B = \begin{pmatrix} \pi & 0 & 0 \\ 0 & 47 & 29 \\ 0 & 0 & 47 \end{pmatrix}.$$

3. Let
$$R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

   Show that A is not diagonalizable when considered as an element of $M_{2\times 2}(\mathbb{R})$, but is diagonalizable as an element of $M_{2\times 2}(\mathbb{C})$.

4. Let R be as above. Find real numbers a_0, a_1, a_2 such that $a_0 I + a_1 R + a_2 R^2 = 0$.

5. Find a basis for the T-cyclic subspace of R^3 generated by v, and where $v = (1, 0, 0)$ and $T : \mathbb{R}^3 \to \mathbb{R}^3$ is given by

$$T(x, y, z) = (x - y + z, x + 2y - z, 3z).$$

   Find the characteristic polynomial of T restricted to this subspace.