Math 110, Fall 2012, Sections 109-110
Worksheet 3

1. Let V be a vector space over $\mathbb{R}$ and $\{v_1, v_2, \ldots, v_m\} \subset V$. List two or three differences between $\{v_1, v_2, \ldots, v_m\}$ and $\operatorname{span}\{v_1, v_2, \ldots, v_m\}$.

2. Let V be a vector space, and suppose $v_1, \ldots, v_k \in V$. What is $\dim \operatorname{span}\{v_1, \ldots, v_k\}$?

3. (a) I'm thinking of a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$. All I'll tell you is that $T(1, 1, 1) = (4, 7)$ and $T(1, 0, -1) = (-2, 3)$. Compute $T(4, 2, 0)$.

   (b) More generally, suppose if I have a linear transformation $T : V \to W$, and I tell you $T(v_1)$, $T(v_2)$, ..., and $T(v_k)$. For which $v \in V$ can you calculate $T(v)$?

4. Give an example of two vectors spaces V and W, and sets of vectors $\{v_1, v_2, v_3\} \subset V$, and $\{w_1, w_2, w_3\} \subset W$ such that there is *no* linear transformation with $T(v_i) = w_i$ for all i.

5. Suppose that T is a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ with $R(T) \subseteq N(T)$.

   (a) What are the possible values of $r(T)$?

   (b) What is $T(T(x))$ for $x \in \mathbb{R}^n$?

6. Suppose $T : V \to W$ is a linear transformation and that $\{v_1, \ldots, v_k\}$ spans V.

   (a) Give an example where $\{T(v_1), \ldots, T(v_k)\}$ does not span W.

   (b) Prove that if $R(T) = W$, then $\{T(v_1), \ldots, T(v_k)\}$ spans W.