Math 1B, Fall 2008
Section 108

Quiz 10 Solutions

(1) *Find*

$$\int \frac{y}{y^2-1}\ dy.$$

Solution 1: Substitute $u = y^2 - 1$ to get

$$\begin{aligned}\int \frac{y\ dy}{y^2-1} &= \frac{1}{2}\int \frac{du}{u} \\ &= \frac{1}{2}\ln\left|y^2-1\right| + C.\end{aligned}$$

Solution 2: Factoring $y^2 - 1 = (y-1)(y+1)$, use partial fractions (calculation omitted) to get

$$\frac{y}{y^2-1} = \frac{1}{2(y+1)} + \frac{1}{2(y-1)}.$$

So

$$\int \frac{y\ dy}{y^2-1} = \int \frac{1}{2(y+1)} + \frac{1}{2(y-1)} = \frac{1}{2}\left(\ln|y+1| + \ln|y-1|\right) + C$$

These two answers are the same, becase

$$\ln\left|y^2-1\right| = \ln\left(|y+1|\,|y-1|\right) = \ln|y+1| + \ln|y-1|\,.$$

(2) *Solve as an implicit function of $y$ and $\theta$:*

$$\frac{dy}{d\theta} = \frac{e^y \sin^2\theta}{y\sec\theta}.$$

Separating variables gives

$$\frac{y\ dy}{e^y} = \frac{\sin^2\theta\ d\theta}{\sec\theta}.$$

Now integrate each side separately

$$\begin{aligned}\int \frac{y\ dy}{e^y} &= \int ye^{-y}\ dy \\ &= -ye^{-y} - \int -e^{-y}\ dy \\ &= -ye^{-y} - e^{-y} \\ &= -(y+1)e^{-y}.\end{aligned}$$

and also (using the subtitution $u = \sin\theta$)

$$\begin{aligned}\int \frac{\sin^2\theta\ d\theta}{\sec\theta} &= \int \sin^2\theta\cos\theta\ dy \\ &= \int u^2\ du \\ &= \frac{1}{3}\sin^3\theta.\end{aligned}$$

So our final answer is

$$-(y+1)e^{-y} = \frac{1}{3}\sin^3\theta + C$$

(3) *A force of 10 lb is needed to stretch a spring 4 inches from its rest length. How much work is done in stretching it 6 inches from rest length?*

When holding a spring stretched, the required force is $F = kx$, so the first fact tells us 10 lb $= k(4$ in), or $k = \frac{5}{2}$ lb/in. So the work to stretch 6 inches from rest length is

$$W = \int_0^6 \frac{5}{2}x\ dx,$$

where $x$ is the amount of extension from rest, in inches. Evaluating the integral, we get that $W = 45$ in·lb. Alternatively, we could convert to feet and get that $k = 30$ lb/ft, and

$$W = \int_0^{\frac{1}{2}} 30x\ dx,$$

which yields $W = 15/4$ ft·lb.