Math 1B, Fall 2008
Section 107

Quiz 1 Solutions

(1) *Find* $\int xe^{-x}dx$.

Use integration by parts:

$$\begin{aligned}\int xe^{-x}dx &= -xe^{-x} - \int -e^{-x}dx \\ &= -xe^{-x} - e^{-x} + C \\ &= -(x+1)e^{-x} + C\end{aligned} \qquad \left|\begin{array}{cc} u = x & dv = e^{-x}dx \\ du = dx & v = -e^{-x}\end{array}\right|$$

(2) *Suppose* $f(x)$ *is continuous on* $[-1,1]$. *Find* $\lim_{x\to\infty} \frac{f(\sin(x))}{x}$.

First a formal solution: Since $-1 \le \sin(x) \le 1$, and $f(x)$ is continuous on $[-1,1]$, the Extreme Value Theorem says that there are numbers m and M such that $m \le f(\sin(x)) \le M$. Since we are considering $x > 0$, we can divide the inequality by x to get

$$\frac{m}{x} \le \frac{f(\sin(x))}{x} \le \frac{M}{x}. \qquad (x > 0)$$

Since $\frac{m}{x} \to 0$ and $\frac{M}{x} \to 0$ as $x \to \infty$, the Squeeze Theorem (Stewart p. 41) says that $\frac{f(\sin(x))}{x} \to 0$ as $x \to \infty$.

A less formal summary: Since $-1 \le \sin(x) \le 1$, and $f(x)$ is continuous on $[-1,1]$, the Extreme Value Theorem says that $f(sin(x))$ stays small even as x gets big. Thus

$$\begin{aligned}\lim_{x\to\infty} \frac{f(\sin(x)}{x} &= \lim_{x\to\infty} \frac{\text{small}}{x} \\ &= 0.\end{aligned}$$

(3) *Find* $\int \sin\sqrt{x}\ dx$.

First a substitution:

$$\int \sin\sqrt{x}\ dx = 2\int r\sin r\ dr \qquad \left|\begin{array}{c} r = \sqrt{x} \\ dr = \dfrac{dx}{2\sqrt{x}} = \dfrac{dx}{2r} \\ dx = 2r\ dr \end{array}\right|$$

Now integrate by parts:

$$\begin{aligned} 2\int r\sin r\ dr &= 2\left(-r\cos r - \int -\cos r dr\right) \\ &= 2\left(-r\cos r + \int \cos r dr\right) \\ &= 2\left(-r\cos r + \sin r\right) + C \\ &= 2\left(\sin\sqrt{x} - \sqrt{x}\cos\sqrt{x}\right) + C \end{aligned} \qquad \left|\begin{array}{cc} u = r & dv = \sin r dr \\ du = dr & v = -\cos r \end{array}\right|$$