Math 1B, Fall 2008
Section 108

<div align="center">Quiz 2 Solutions</div>

(1) *Find* $\int \dfrac{1}{\sqrt{t^2 - 6t + 13}}\, dt.$

First complete the square: $t^2 - 6t + 13 = (t-3)^2 + 4$. Now substitute
$\begin{bmatrix} u = t - 3 \\ du = dt \end{bmatrix}$ to get

$$\int \frac{1}{\sqrt{t^2 - 6t + 13}}\, dt \;=\; \int \frac{1}{\sqrt{(t-3)^2 + 4}}\, dt$$
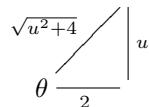$$=\; \int \frac{1}{\sqrt{u^2 + 4}}\, du.$$

This is now a trig substitution problem. We will make the substitution

$$\begin{bmatrix} u = 2\tan\theta & -\pi/2 < \theta < \pi/2 \\ du = 2\sec^2\theta\, d\theta \end{bmatrix},$$

and use the fact that $\sqrt{1 + \tan^2\theta} = \sqrt{\sec^2\theta}$ which is equal to $\sec\theta$ on the chosen domain.

$$\int \frac{1}{\sqrt{u^2 + 4}}\, du \;=\; \int \frac{2\sec^2\theta}{\sqrt{4\tan^2\theta + 4}}\, d\theta$$
$$=\; \int \frac{2\sec^2\theta}{2\sqrt{\tan^2\theta + 1}}\, d\theta$$
$$=\; \int \frac{\sec^2\theta}{\sec\theta}\, d\theta$$
$$=\; \int \sec\theta\, d\theta$$
$$=\; \ln|\sec\theta + \tan\theta|.$$

Reversing our substitution, $\tan\theta = u/2 = (t-3)/2$. To find $\sec\theta$, we use the triangle

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and we can read off $\sec\theta = \dfrac{\sqrt{u^2+4}}{2} = \dfrac{\sqrt{(t-3)^2+4}}{2}$. So putting it all together we get

$$\int \frac{1}{\sqrt{t^2-6t+13}}\,dt = \ln\left|\frac{\sqrt{(t-3)^2+4}}{2} + \frac{t-3}{2}\right| + C$$

This can optionally be simplified to

$$\ln\left|\sqrt{(t-3)^2+4}+t-3\right| + C_2.$$

(Why?)

(2) *Find* $\int \dfrac{\tan(1/z)}{z^2}\,dz.$

Using the subtitution $\begin{bmatrix} u = \dfrac{1}{z} \\ du = -\dfrac{1}{z^2}dz \end{bmatrix}$ we get

$$
\begin{aligned}
\int \frac{\tan(1/z)}{z^2}\,dz &= -\int \tan u\,du \\
&= -\ln|\sec u| + C \\
&= -\ln\left|\sec\frac{1}{z}\right| + C \\
&= \ln\left|\cos\frac{1}{z}\right| + C
\end{aligned}
$$

Remarks: The integral $\int \tan u\,du$ is on the cheat sheet. Both $-\ln\left|\sec\dfrac{1}{z}\right|$ and $\ln\left|\cos\dfrac{1}{z}\right|$ are acceptable answers.

(3) *Find* $\int p^5 \ln p\,dp$

We use the following data to integrate by parts:

$$\begin{bmatrix} u = \ln p & dv = p^5\,dp \\ du = \dfrac{1}{p}\,dp & v = \dfrac{1}{6}p^6 \end{bmatrix}$$

$$
\begin{aligned}
\int p^5 \ln p\,dp &= \frac{1}{6}p^6 \ln p - \int \frac{1}{6}p^5\,dp \\
&= \frac{1}{6}p^6 \ln p - \frac{1}{36}p^6 + C
\end{aligned}
$$

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