Math 1B, Fall 2008
Section 108

Quiz 3 Solutions

(1) *Find* $\int_1^\infty \frac{dx}{(3x+1)^2}$.

First find the antiderivitive, using $u = 3x + 1$,

$$\begin{aligned}\int \frac{dx}{(3x+1)^2} &= \frac{1}{3}\int \frac{du}{u^2} \\ &= -\frac{1}{3}u^{-1} \\ &= \frac{-1}{3(3x+1)}\end{aligned}$$

Now evaluate the improper integral

$$\begin{aligned}\int_1^\infty \frac{dx}{(3x+1)^2} &= \lim_{t\to\infty}\int_1^t \frac{dx}{(3x+1)^2} \\ &= \lim_{t\to\infty} -\frac{1}{3(3x+1)}\Big]_1^t \\ &= \lim_{t\to\infty} \frac{1}{12} - \frac{1}{3(3t+1)} \\ &= \frac{1}{12} - 0 = \frac{1}{12}.\end{aligned}$$

(2) *Find* $\int \frac{r^2}{r+4}\,dr$.

Solution 1: Use polynomial long devision to find

$$\frac{r^2}{r+4} = r - 4 + \frac{16}{r+4}.$$

Thus

$$\begin{aligned}\int \frac{r^2}{r+4}\,dr &= \int r - 4 + \frac{16}{r+4}\,dr \\ &= \frac{1}{2}r^2 - 4r + 16\ln|r+4| + C\end{aligned}$$

Solution 2: Substitute $u = r + 4$ (which gives $du = dr$ and $u = r - 4$). Now we can calculate

$$\begin{aligned}
\int \frac{r^2}{r+4}\,dr &= \int \frac{(u-4)^2}{u}\,du \\
&= \int \frac{u^2 - 8u + 16}{u}\,du \\
&= \frac{1}{2}u^2 - 8u + 16\ln|u| + C \\
&= \frac{1}{2}(r+4)^2 - 8(r+4) + 16\ln|r+4| + C.
\end{aligned}$$

This answer doesn't look the same as solution 1, but if you expand it out, they only differ by a constant.

(3) *How large must we choose n so that $|E_L| \leq \frac{1}{7}$ in evaluating $\int_0^1 x^2\,dx$?*

We know $|E_L| \leq \dfrac{K_1(b-a)^2}{2n}$, and we want $|E_L| \leq \frac{1}{7}$, so we find n big enough that $\dfrac{K_1(b-a)^2}{2n} \leq \dfrac{1}{7}$. In this case, $b - a = 1$, so all we need to find is K_1:

$$K_1 = \max_{0\leq x\leq 1} |f'(x)| = \max_{0\leq x\leq 1} |2x| = 2$$

So

$$\frac{K_1(b-a)^2}{2n} = \frac{1}{n}.$$

To find when $\frac{1}{n} \leq \frac{1}{7}$, multiply both sides by $7n$ to get $7 \leq n$. Thus $n = 7$ is the smallest acceptable choice. (Notice that when we took the reciprocal, the direction of the inequality changed.)