Math 1B, Fall 2008
Section 108

Quiz 6 Solutions

(1) *Determine whether the following series converges or diverges,*

$$\sum_{n=1}^{\infty} \frac{n-1}{n4^n}.$$

Thinking that $\frac{n-1}{n} \approx 1$ for large $n$, we apply the Limit Comparison Test with

$$a_n = \frac{n-1}{n4^n} \qquad b_n = \frac{1}{4^n}.$$

The test applies, as $a_n, b_n \geq 0$ and

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n-1}{n} = 1.$$

Since $\sum_{n=1}^{\infty} b_n$ converges (geometric series with $|r| < 1$), $\sum_{n=1}^{\infty} \dfrac{n-1}{n4^n}$ also converges by the Limit Comparison Test.

This could also be done with the ratio test.

(2) *Determine whether the following series is absolutely convergent, conditionally convergent, or divergent,*

$$\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}.$$

We use the ratio test. If $a_n = \dfrac{(-10)^n}{n!}$, then

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{10^{n+1}}{(n+1)!} \cdot \frac{n!}{10^n} = \lim_{n\to\infty} \frac{10}{n+1} = 0.$$

Thus by the ratio test, $\sum_{n=1}^{\infty} \frac{5^{n-1}}{n^2 4^n}$ converges absolutely.

(3) *Determine whether the following series is absolutely convergent, conditionally convergent, or divergent*

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n!}.$$

If $a_n = \frac{\cos(n\pi/3)}{n!}$, then

$$|a_n| \leq \frac{1}{n!}$$

because $|\cos x| \leq 1$ for all real numbers $x$. We now apply the ratio test to $\sum_{n=1}^{\infty} \frac{1}{n!}$. If $b_n = \frac{1}{n!}$, then

$$\lim_{n\to\infty} \left| \frac{b_{n+1}}{b_n} \right| = \lim_{n\to\infty} \frac{1}{n+1} = 0.$$

Thus $\sum_{n=1}^{\infty} b_n$ converges by the Ratio Test, which means that $\sum_{n=1}^{\infty} |a_n|$ converges by the Comparison Test. We conclude that

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n!}$$

is absolutely convergent.