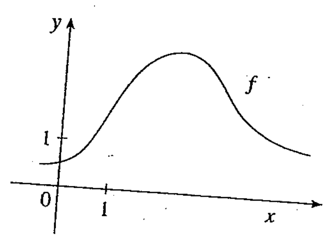
Math 1B, Fall 2008
Sections 107/108

Quiz 8 Solutions

(1)



Say why the following could not be the Taylor series for $f(x)$ centered at a

(i) $a = 1$; $1.6 - 0.8(x-1) + 0.4(x-1)^2 + \cdots$

(ii) $a = 2$; $2.8 + 0.5(x-2) + 1.5(x-2)^2 + \cdots$

(i) If the given expression were the Taylor series for f about 1, then we would have $f'(1) = -0.8$. But f is increasing at $x = 1$, so $f'(1)$ could not be negative.

(ii) The graphed function is concave down near $x = 2$, so we could not have $f''(2) = 1.5 * 2 > 0$.

(2) *Find the value of the sum*

$$\sum_{n=1}^{\infty} \frac{3^n}{4^n n!}.$$

Notice that

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{x^0}{0!} + \sum_{n=1}^{\infty} \frac{x^n}{n!} = 1 + \sum_{n=1}^{\infty} \frac{x^n}{n!},$$

so

$$\sum_{n=1}^{\infty} \frac{x^n}{n!} = e^x - 1.$$

Since this power series converges for all x, we can plug in $x = 3/4$ to get

$$\sum_{n=1}^{\infty} \frac{3^n}{4^n n!} = e^{\frac{3}{4}} - 1.$$

(3) *Find a power series expansion for* $f(x) = \ln(1 + 5x)$.

Becuase

$$\frac{d}{dx} \ln(1 + 5x) = \frac{5}{1 + 5x},$$

we have

$$\begin{aligned}
\ln(1+5x) &= \int \frac{5}{1+5x}\, dx \\
&= 5 \int \sum_{n=0}^{\infty} (-5)^n x^n \, dx \\
&= 5 \sum_{n=0}^{\infty} \int (-5)^n x^n \, dx \\
&= C + \sum_{n=0}^{\infty} \frac{(-1)^n 5^{n+1} x^{n+1}}{n+1} \\
&= C + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 5^n x^n}{n}.
\end{aligned}$$

Because $\ln(1 + 5x)$ is a specific antiderivative of $\frac{5}{1+5x}$, and not the family of *all* antiderivatives, we need to find the appropriate value of C. Plugging in $x = 0$ gives $C = \ln(1) = 0$. So the final answer is

$$\ln(1 + 5x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 5^n x^n}{n}.$$