Math 1B, Fall 2008
Section 108

Quiz 9 Solutions

(1) *Find*

$$\int \frac{1}{\sqrt{x + x^{3/2}}}\ dx.$$

Using the substitution $u = 1 + \sqrt{x}$ (and $2\ du = 1/\sqrt{x}\ dx$), we get

$$\begin{aligned} \int \frac{dx}{\sqrt{x + x^{3/2}}} &= \int \frac{dx}{\sqrt{x}\sqrt{1+\sqrt{x}}} \\ &= 2\int \frac{du}{\sqrt{u}} \\ &= 4\sqrt{u} \\ &= 4\sqrt{1+\sqrt{x}} + C. \end{aligned}$$

(2) *Find the length of the curve* $y = \ln(\sec(x))$ *for* $0 \le x \le \pi/4$.

Since $dy/\ dx = \sec(x)\tan(x)/\sec(x) = \tan(x)$, we have

$$\begin{aligned} L &= \int_0^{\pi/4} \sqrt{1 + \tan^2(x)}\ dx \\ &= \int_0^{\pi/4} \sec x\ dx \\ &= \ln|\sec x + \tan x|]_0^{\pi/4} \\ &= \ln(\sqrt{2} + 1). \end{aligned}$$

(3) *Evaluate the following integral or show that it is divergent.*

$$\int_0^4 \frac{\ln x}{\sqrt{x}}\ dx.$$

First, integrating by parts gives that

$$\int \frac{\ln x}{\sqrt{x}}\ dx = 2\sqrt{x}\ln(x) - 4\sqrt{x}.$$

Since $\lim_{x\to 0} \frac{\ln x}{\sqrt{x}} = \infty$, the definite integral is improper at $x = 0$. So

$$\begin{aligned}
\int_0^4 \frac{\ln x}{\sqrt{x}}\ dx &= \lim_{a\to 0^+} \int_a^4 \frac{\ln x}{\sqrt{x}}\ dx \\
&= \lim_{a\to 0^+} 4\ln 4 - 8 - 2\sqrt{a}\ln a - 4\sqrt{a} \\
&= 4\ln 4 - 8 - 2\lim_{a\to 0^+} \sqrt{a}\ln a.
\end{aligned}$$

This last limit is a $(-\infty)\cdot 0$ indefinite form, so we use L'Hopital's rule:

$$\begin{aligned}
\lim_{a\to 0^+} \sqrt{a}\ln a &= \lim_{a\to 0^+} \frac{\ln a}{a^{-1/2}} \\
&= \lim_{a\to 0^+} \frac{1/a}{-\frac{1}{2}a^{-3/2}} \\
&= \lim_{a\to 0^+} -2\sqrt{a} = 0.
\end{aligned}$$

So we conclude that

$$\int_0^4 \frac{\ln x}{\sqrt{x}}\ dx = 4\ln 4 - 8.$$