Math 32, Spring 2010, Section 101
Worksheet 10

Work through the following problems in groups of about four. Take turns writing; everyone should get a chance to write for some of the problems. It's more important to understand the problems than to do all of them.

1. Evaluate $\sin(7\pi/2)$ and $\sin(-7\pi/2)$.

   Since $7\pi/2 = 2\pi + 3\pi/2$, we have $\sin(7\pi/2) = \sin(3\pi/2) = -1$. This tells us that $\sin(-7\pi/2) = -\sin(7\pi/2) = 1$.

2. The point $(16\pi/3, y)$ is on the graph of $y = \sin x$. Can you determine the value of y? What can you say about it?

   For the given point to be on the graph of $y = \sin x$, we must have $y = \sin(16\pi/3) = \sin(10\pi/3) = \sin(4\pi/3) = -\sqrt{3}/2$.

3. The point $(x, \frac{1}{2})$ is on the graph of $y = \cos x$. Can you determine the value of x? What can you say about it?

   We must have $\sin x = \frac{1}{2}$. However, this doesn't determine the value of x. For example, the angles $\pi/6$ and $5\pi/6$ are both possible values of 6. In fact, we could have $x = 2\pi n + \pi/6$ or $x = 2\pi n + 5\pi/6$ for any integer n.

4. Prove that the following equation is an identity: $\sin^2 t - \cos^2 t = \dfrac{1 - \cot^2 t}{1 + \cot^2 t}$.

$$\begin{aligned} \frac{1 - \cot^2 t}{1 + \cot^2 t} &= \frac{1 - \frac{\cos^2 t}{\sin^2 t}}{1 + \frac{\cos^2 t}{\sin^2 t}} \\ &= \frac{\sin^2 t - \cos^2 t}{\sin^2 t + \cos^2 t} \\ &= \sin^2 t - \cos^2 t. \end{aligned}$$

5. Use the unit circle to justify the fact that $\sin(t + \pi) = -\sin t$. Graph $y = \sin t$ and explain how this shows up on the graph.

6. It is a fact that $\sin 2 \approx 0.909$. Find another positive number and a negative number whose sines are also ≈ 0.909.

   Possible options are: $2 + 2\pi$, $2 - 2\pi$, etc.