Math 32, Spring 2010, Section 101
Worksheet 3 Solutions

Work through the following problems in groups of about four. Take turns writing; everyone should get a chance to write for some of the problems. It's more important to understand the problems than to do all of them.

1. Find the center and radius of the circles determined by the following equations.

   (a) $(x-1)^2+(y+2)^2=9$. Th equation is in standard form, so we can just read off the center and radius. The center is $(1,-2)$ and the radius is $\sqrt{9}=3$. For more, see p.63.

   (b) $x^2+y^2-10x+2y+17=0$. We need to complete the square for both the x terms and the y terms. Let's rewrite this as $(x^2-10x)+(y^2+2y)=-17$. To make the x terms into something of the form $(x-h)^2$, we need to add 25 to both sides. We then have $(x^2-10x+25)+(y^2+2y)=-17$. That is, $(x-5)^2+(y^2+2y)=-17$. Next, we add 1 to both sides to complete the square for the y terms, and we get $(x-5)^2+(y+1)^2=-17+25+1=9$. So the center is $(5,-1)$ and the radius is 3. For more, start reading at the bottom of p.64, or come to office hours.

2. If there are any, find the y-intercept(s) of the circles from question 1. Also, determine if the point $(4,-2)$ is on each circle.

   (a) We find the y-intercepts by looking for solutions to the equation (i.e. points on the circle) where $x=0$. That is, we want to know which y values give us $(0-1)^2+(y+2)^2=9$. Rearranging the equation, this is equivalent to $1+y^2+4y+4=9$, or $y^2+4y-4=0$. The quadratic formula says that the solutions are $\frac{-4\pm\sqrt{16+16}}{2}=-2\pm2\sqrt{2}$. These are the y-intercepts. To test if $(4,-2)$ is on the circle, we can simply plug it into the equation and see if it is true. In this case $(4-1)^2+(-2+2)^2=9$, so the point is on the circle.

   (b) Plugging $x=0$ into the original equation (the standard equation we derived would work, but this is simpler) gives $y^2+2y+17=0$. The discriminant $4-4*17$ is negative, so this equation has no real solutions. That means this circle has no y-intercepts (try graphing it to see if this make sense!). To see if $(4,-2)$ is on the circle, we can plug in and check $(4-5)^2+(-2+1)^2\neq 9$, so the point is not on the circle.

3. How many (real) solutions do the following quadratic equations have? (Hint: you don't have to do all of the work to find them.)

   (a) $2x^2 - 10x + 5 = 0$. We just need to check the discriminant (see p.87). In this case, $(-10)^2 - 2 * 4 * 5 = 60$ is positive, so the equation has two real solutions.

   (b) $\sqrt{2}y^2 + \sqrt{3}y + 1 = 0$. The discriminant is $3 - 4\sqrt{2}$. Since $\sqrt{2}$ is bigger than 1, $4 * \sqrt{2}$ is bigger than 4. Thus $3 - 4\sqrt{2}$ is negative, and there are no real solutions.

   (c) $t^2 - 2t = -1$. In standard form $t^2 - 2t + 1 = 0$. The discriminant is 0, so it has one real solution.

4. Solve the following equations. When appropriate, check for extraneous solutions.

   (a) $|x - 4| - 5 = 2$. Rearranging gives $|x - 4| = 7$. That means that $x - 4 = 7$ or $x - 4 = -7$. In the first case, $x = 11$, and in the second $x = -3$.

   (b) $x^4 - 5x^2 = -6$. Substituting $t = x^2$ gives the equivalent equation $t^2 - 5t + 6 = 0$. Factoring, we get $(t - 2)(t - 3) = 0$, so $t = 2$ or $t = 3$. Since $t = x^2$, this tells us that $x^2 = 2$ or $x^2 = 3$. These are equivalent to $x^2 - 2 = 0$ and $x^2 - 3 = 0$. These quadratics have solutions $x = \pm\sqrt{2}$ and $x = \pm\sqrt{3}$, which are the solutions to the original equation.

   (c) $8t^{-2} - 17t^{-1} + 2 = 0$. We could substitute $y = t^{-1}$ and proceed as in the previous problem, but instead let's multiply both sides by t^2. This gives us $8 - 17t + 2t^2 = 0$. Applying the quadratic formula, we get solutions $t = \frac{1}{4}(17 \pm \sqrt{17^2 - 4 * 2 * 8}$, or $t = \frac{1}{2}$ and $t = 8$. Since we multiplied both sides by an equation involving t, we need to check for extraneous solutions. Go ahead and check!

   (d) $\sqrt{2 - x} - 10 = x$. Since there is only one square root, we can isolate it. That is, let's add 10 to both sides to get $\sqrt{2 - x} = x + 10$. We can now square both sides to get $2 - x = (x + 10)^2 = x^2 + 20x + 100$. Adding $x - 2$ to both sides gives $x^2 + 21x + 98 = 0$. Doing the quadratic formula (probably with a calculator; on quizzes and tests, the numbers will usually be smaller), we get solutions $x = -14$ and $x = -7$. Since we squared both sides, we need to check for extraneous solutions. $x = -7$ works, but plugging in $x = -14$ involves taking the square root of a negative number, so this is not a valid solution.

5. Find the value(s) of k such that $kx^2 + kx + 1 = 0$ has exactly one real solution.

This will have exactly one real solution when the discriminant is 0, assuming the coefficient of x^2 is not 0 (if the coefficient of x^2 is 0, it's no longer quadratic, and all bets are off!). The disciminant is $k^2 - 4k$, so we're looking for solutions to $k^2 - 4k = 0$. Factoring, we get $k(k-4) = 0$ which has solutions $k = 0$ and $k = 4$. However, as we noted $k = 0$ makes the equation no longer quadratic, and testing we can indeed see that $0x^2 + 0x + 1 = 0$ actually has no real solutions (it's just $1 = 0$, which is impossible). So in this case, $k = 4$ is the only value that will give you exactly one real solution.