Math 32, Spring 2010, Section 101
Worksheet 8

Work through the following problems in groups of about four. Take turns writing; everyone should get a chance to write for some of the problems. It's more important to understand the problems than to do all of them.

1. Assume that the population of a bacteria colony grows exponentially (i.e. according to the law $N(t) = N_0 e^{kt}$.) At the start of an experiment, 2000 bacteria are present in a colony. Eight hours later, the population is 3000.

   (a) Determine the constants N_0 and k in the model.

   (b) What was the population two hours after the start of the experiment?

   (c) How long will it take for the population to triple?

   (a) Plugging in $t = 0$ gives $N_0 = 2000$. Plugging in $N(8) = 3000$, we get $3000 = 2000e^{8k}$. Dividing by 2000 gives $3/2 = e^{8k}$. Taking natural log, we get $\ln 3/2 = 8k$, or $k = \frac{1}{8}\ln(3/2)$.

   (b) Two hours later, we have
   $$N(2) = 2000e^{2k} = 2000e^{\frac{1}{4}\ln(3/2)} = 2000\left(e^{\ln 3/2}\right)^{\frac{1}{4}} = 2000\,(3/2)^{\frac{1}{4}}.$$

   (c) We are trying to find t for which $6000 = 2000e^{\frac{1}{8}\ln(3/2)t}$. Dividing, we get $3 = e^{\frac{1}{8}\ln(3/2)t}$. Taking natural log, we get $\ln 3 = \frac{1}{8}\ln(3/2)t$. Rearranging, this gives $t = 8\ln 3/\ln(3/2)$. Alternatively, this is $8\log_{3/2}(3)$.

2. Given that β is an acute angle and that $\sin\beta = 2/5$, find the values of the other five trigonometric functions at β.

   One way to solve this is to draw a right triangle, with an angle β. The fact that $\sin\beta = 2/5$ means that we can label the leg opposite β to have length 2, and the hypotenuse to have length 5. The pythagorean theorem then says that the adjacent leg has length $\sqrt{21}$. Using our right triangle trig formulas, we now have $\cos\beta = \sqrt{21}/5$ and $\tan\beta = 2/\sqrt{21} = 2\sqrt{21}/21$. To get the final three values, we just have to take reciprocals. To get secant, we compute $\sec\beta = 1/\cos\beta = 5/\sqrt{21} = 5\sqrt{21}/21$. We also have $\csc\beta = 5/2$ and $\cot\theta = \sqrt{21}/2$.

3. Suppose the points A, B and C form a right triangle, with the right angle at point C. Suppose the angle A is 60 degrees and that $AB = 12$cm. Find AC and BC.

   Noting that AB is the hypotenuse, our trig formulas give $\cos 60 = AC/AB$. Solving for AC, we get $AC = \cos 60 * AB$. We were given $AB = 12$, and we know $\cos 60 = 1/2$. Thus $AC = 12 * 1/2 = 6$. Similarly, $BC = \sin 60 * AB = 12 * \sqrt{3}/2 = 6\sqrt{3}$.

4. The element WillieNelsonium-32 is observed to decay according to the law $N(t) = N_0 e^{47t}$. What is the half-life of WillieNelsonium-32?

   We want to know the time for which $\frac{1}{2}N_0 = N_0 e^{47t}$. Dividing by N_0, we get $\frac{1}{2} = e^{47t}$, or $\ln(1/2) = 47t$. Rearranging, this gives the half-life to be $\frac{1}{47}\ln(1/2)$.

5. Explain with pictures why $\sin\theta \leq 1$ and $\cos\theta \leq 1$, but $\tan\theta$ can potentially be really big or really small.

   Draw a right triangle, and label an angle θ. Notice that both of the legs are shorter than the hypotenuse (this is confirmed by the Pythagoren theorem: since $a^2 + b^2 = c^2$, we must have that c is bigger than a or b; otherwise, $a^2 + b^2$ would be bigger than c^2). Since $\cos\theta$ = adjacent leg / hypotenuse, and the adjacent leg is shorter than the hypotenuse, $\cos\theta \leq 1$. Similarly, $\sin\theta$ = opposite leg/hypotenuse is also ≤ 1. However, $\tan\theta$ = opposite/adjacent. Notice that if θ is really small, then the opposite leg is really short, and the adjacent leg is really big. When you divide a small number by a big number, you get a really small number. Thus if θ is close to 0, $\tan\theta$ is really close to 0. On the other hand, if θ is close to $90°$, then the opposite leg is really short, and the adjacent leg is really long. Thus $\tan\theta$ is a really big number divided by a really small number, which is a really really big number.