Math 54, Spring 2009, Sections 109 and 112
Differential Equations Review

The purpse of this sheet is to outline some of the major types of problems we covered in our study of differential equations. This is a way to start your review, but for a more complete picture it would be best to go back through old homework problems.

Find a formula for the general solution to:

- ODEs of the form $ay'' + by' + c = 0$ (Sections 4.2-4.3)
- ODEs of the form $ay'' + by' + c = f(x)$ for certain kinds of functions $f(x)$. Use the method of undetermined coefficients and the superposition principle(Sections 4.4-4.5)
- ODEs of the form $a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$. This requires finding the roots of the polynomial $a_n r^n + \cdots + a_0$, which will either be given to you or can be found with guess/check and polynomial long division (Section 6.2)
- systems of first-order ODEs with constant coefficients of the form $\vec{x}' = A\vec{x}$ (Sections 9.5-9.6)
- heat flow problems $u_t = \beta u_{xx}$, with initial condition $u(x,0) = f(x)$ and various boundary conditions. For instance, homogenous boundary values for u ($u(0,t) = u(L,t) = 0$), homogenous boundary values for u_x, or non-zero constant boundary values for u (Sections 10.2 and 10.5)
- wave equation problems $u_{tt} = \alpha^2 u_{xx}$ with initial conditions and homogenous boundary conditions (Sections 10.2 and 10.6)

Any of the ODEs listed above could come with initial or boundary conditions, which one would then plug into the general solutions and solve for the appropriate values of the parameters.

Solvability of initial value problems without a formula:

- Solvability of initial value problems for linear ODEs with continuous data (Section 6.1)

- Solvability of initial value problems for systems of first-order linear ODEs with continuous data (Section 9.4)

You should also know how to:

- transform systems of linear ODEs and higher order ODEs into a matrix system in normal form (Section 9.1)
- determine if a set of functions is linearly independent. If all of the functions solve a linear ODE, then you can use the Wronksian (Sections 6.1 and 9.1)
- compute the Fourier series of a function whose domain is $[-T, T]$, and the Fourier sine and cosine series of a function whose domain is $[0, T]$ (Sections 10.3-10.4)
- graph and write a formula for the function a Fourier series converges to (Sections 10.3-10.4)
- use separation of variables to turn a PDE into a system of ODEs, and then turn these ODEs into a set of fundamental solutions (Section 10.2)