Math 54, Spring 2009, Sections 109 and 112
Midterm 1 Review Exercises

These exercises don't cover some of the **very important** computational-type problems, including many of the things listed under the "be able to" section of the review sheet. You can find examples of those types of problems on the sample exam and in the sections of the book (including the supplemental exercises at the end of each chapter). These are a little more theoretical, and are aimed at making sure you have a good grasp of the ideas underlying the algorithms.

1) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and A be its standard matrix. Complete the following table:

Property of T	Columns of A	Pivots of A	$A\vec{x} = \vec{b}$?
One-to-one			≤ 1 solution for every $\vec{b}$
	Span $\mathbb{R}^m$		
		Every row and column ($n = m$)	

2) Let A be a 17×17 matrix such that $A^{12} = I_{17}$. What can you say about Rank(A)? Nul(A)? Find a basis for Col(A).

3) (#10, p.184) Suppose A is invertible. Explain why $A^T A$ is also invertible, and then show that $A^{-1} = (A^T A)^{-1} A^T$.

4) Suppose you have a square matrix such that $A^3 = 0$ (the zero matrix). Use matrix algebra to compute $(I - A)(I + A + A^2)$. Generalize to show that if $A^k = 0$ for some $k \geq 1$, then $(I - A)$ is invertible.

5) True or False? If true, justify. If false, provide a counterexample. (Some of these are from p.102.)

(a) If $\{\vec{v}_1, \vec{v}_2\}$ is a linearly independent set in $\mathbb{R}^n$, so is $\{\vec{v}_1, \vec{v}_1 + \vec{v}_2\}$.

(b) If an $m \times n$ matrix A has a pivot in every column or has a pivot in every row, then it is invertible.

(c) If T is a linear transformation, then $T(\vec{0}) = \vec{0}$.

(d) If A is a square matrix, then it can be written as a product of elementary matrices.

(e) If A is an $n \times n$ matrix such that $A\vec{x} = \vec{b}$ is consistent for every $\vec{b}$, then A has a pivot in every column.

Bonus: **6)** If A is $m \times n$ and B is $n \times r$, then both $\operatorname{Col} B$ and $\operatorname{Nul} A$ are subspaces of $\mathbb{R}^n$. If it happens that $\operatorname{Col} B \subseteq \operatorname{Nul} A$, find the matrix product AB.