Math 54, Spring 2009, Sections 109 and 112
Worksheet 3 (Lay 2.8-2.9)

(1) True or False? If true, justify. If false, give a counterexample.

(a) A set of 3 vectors in $\mathbb{R}^4$ must be linearly independent.

(b) A set of 3 vectors in $\mathbb{R}^4$ cannot span $\mathbb{R}^4$.

(c) A set of 5 vectors in $\mathbb{R}^4$ must be linearly dependent.

(d) A set of 5 vectors in $\mathbb{R}^4$ must span $\mathbb{R}^4$.

(2) Let $H \subseteq \mathbb{R}^n$ be a subspace. What is the definition of a basis for H? What is the definition of the dimension of H?

(3) Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -2 & 4 \\ 1 & 4 & -2 \end{bmatrix}$. Find a basis for $\operatorname{Col} A$. What is $\dim \operatorname{Col} A$? What is $\dim \operatorname{Nul} A$?
(Hint: you don't need to do any extra work to answer the last part.)

(4) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and let $H = \operatorname{Span}\{v_1, v_2\} \subseteq \mathbb{R}^3$. Find the coordinates of $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ with respect to the basis $\mathfrak{B} = \{v_1, v_2\}$ for H.