Name: ______________________________

Math 54, Summer 2009, Lecture 4
Quiz 10 Solution

(1) For each ODE, tell whether or not it is linear (and if not, why). If it is linear, give the largest interval on which you can guarantee the given IVP has a solution.

(a) $(t+1)y''' - (t+2)y'' - (t+4)y = e^t + \cos t; \quad y(0) = 1, \quad y'(0) = -1, \quad y''(0) = 47.$

It is linear. In standard form, it becomes

$$y''' - \frac{(t+2)}{(t+1)}y'' - \frac{(t+4)}{(t+1)}y = \frac{1}{(t+1)}(e^t + \cos t).$$

The largest interval containing 0 on which all of the functions are continuous is $(-1, \infty)$.

(b) $y''' - (2y')^2 + 3y = 0, \quad y(3) = 1, \quad y'(3) = 0, \quad y''(3) = 4$

It is not linear. You can tell because of the $(y')^2$ term. More rigorously, one could check that $L[-y] \neq -L[y]$.

(c) $\cos(t)y'' - 2y' + y = \ln(t), \quad y(1) = 19, \quad y'(1) = 19$

It is linear. In standard form:

$$y'' - \frac{2}{\cos(t)}y' + \frac{1}{\cos(t)}y = \frac{\ln(t)}{\cos(t)}.$$

Since $\cos(t) = 0$ when $t = \pi/2 + \pi k$ and $\ln(t)$ is only defined for $t > 0$, the largest interval on which we may guarantee a solution is $(0, \pi/2)$.

(2) Solve the initial value problem $y'' + y' - 6y = 0, \quad y(0) = -1, \quad y'(0) = 7.$

The auxiliary equation is $0 = r^2 + r - 6 = (r+3)(r-2)$. Thus the general solution to the ODE is $c_1 e^{-3t} + c_2 e^{2t}$. Plugging in the initial conditions we get

$$\begin{aligned} -1 &= c_1 + c_2, \\ 7 &= -3c_1 + 2c_2. \end{aligned}$$

Solving gives $c_1 = -\frac{9}{5}$ and $c_2 = \frac{4}{5}$, so the solution to the IVP is $-\frac{9}{5}e^{-3t} + \frac{4}{5}e^{2t}$.

(2) If possible, find all solution(s) to the boundary value problem $y'' + \pi^2 y = 0, \quad y(0) = 1,$ $y(2) = 1$.

The auxiliary equation is $r^2 + \pi^2 = 0$, which has roots $\pm \pi i$. Thus the general solution to the ODE is $c_1 \cos \pi t + c_2 \sin \pi t$. Plugging in the initial conditions we get

$$\begin{aligned} 1 &= c_1, \\ 1 &= c_1. \end{aligned}$$

Hence there is no requirement on c_2, and the solutions to this BVP are the functions $\cos \pi t + c_2 \sin \pi t$, where c_2 can be any real number.