Math 54, Summer 2009, Lecture 4
Worksheet 1: Lay 1.7

(1) Classify the following sets as linearly independent or linearly dependent. (Hint: many don't require calculation).

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right\}$.

(b) $\left\{ \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} \right\}$.

(c) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \end{bmatrix} \right\}$.

(d) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(e) $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ -6 \\ 9 \\ -12 \end{bmatrix} \right\}$.

(2) True or False: The columns of a matrix A are linearly dependent if and only if the equation $A\vec{x} = \vec{0}$ is consistent. Justify your answer.

(3) Suppose $\vec{v}_1, \ldots, \vec{v}_4$ are vectors in $\mathbb{R}^3$. Let $S_2 = \{\vec{v}_1, \vec{v}_2\}$, $S_3 = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, and $S_4 = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$. For each of the following, mark the statement true or false. As always, justify your answer.

(a) If $\operatorname{Span} S_4 = \mathbb{R}^3$, then $\operatorname{Span} S_3 = \mathbb{R}^3$.

(b) If $\operatorname{Span} S_3 = \mathbb{R}^3$, then $\operatorname{Span} S_4 = \mathbb{R}^3$.

(c) If S_2 is linearly dependent, then so is S_3.

(d) If S_3 is linearly dependent, then so is S_2.