

**Math 54, Summer 2009, Lecture 4**  
**Worksheet 3: Lay 4.3**

Let  $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 3 & -2 \\ 3 & 1 & 4 \end{bmatrix}$ . In a group, use the following ideas to find a basis for  $\text{Col } A$ .

- a) Explain why the pivot columns of an echelon form of  $A$  are linearly independent, and why no bigger set of columns containing the pivot columns is linearly independent (Hint: think of about pivots in the matrix formed by the set of columns).
  
- b) Explain why echelon forms of  $A$  can be factored  $PA$ , where  $P$  is invertible.
  
- c) Show that for any  $\vec{x} \in \mathbb{R}^3$ , we have  $A\vec{x} = \vec{0}$  if and only if  $PA\vec{x} = \vec{0}$ , and use this to show that a set of columns in  $A$  is linearly independent if and only if the corresponding columns in echelon form are linearly independent.
  
- d) What is a basis for  $\text{Col } A$ ?