Base case: When $n=1$, we have $3 = 4 \cdot 1^2 - 1$.

Inductive step: Suppose $3 + 11 + \ldots + (8n-5) = 4(n^2 - n)$.

Then $3 + 11 + \ldots + (8n-5) + (8(n+1)-5) = 4(n^2 - n) + (8(n+1)-5)$.

$= 4n^2 - n + 8n + 3 = 4(n^2 + 2n + 1) - (n + 1) = 4(n+1)^2 - (n+1)$

which completes the proof.

Base case: When $n=1$, we have $1 + \frac{1}{2} = 2 - \frac{1}{2}$.

Inductive step: Suppose $1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n} = 2 - \frac{1}{2^n}$.

Then $1 + \frac{1}{2} + \ldots + \frac{1}{2^n} + \frac{1}{2^{n+1}} = 2 - \frac{1}{2^n} + \frac{1}{2^{n+1}}$

$= 2 + (\frac{1}{2^{n+1}} - \frac{2}{2^{n+1}}) = 2 - \frac{1}{2^n}$, which completes the proof.

Base case: When $n=2$, we have $2^2 = 4 > 3 = 2 + 1$.

Inductive step: Suppose $n^2 > n+1$. Then

$(n+1)^2 = n^2 + 2n + 1 > (n+1) + 2n + 1 = (n+1) + 1$,

which completes the proof.

Base case: When $n=4$, we have $4! = 24 > 16 = 4^2$.

Inductive step: Suppose $n! > n^2$. Then

$(n+1)! = (n+1) \cdot n! > (n+1)n^2$. By part (a),

$n^2 > n+1$ for all $n \geq 2$. Therefore, we may continue the previous inequality to obtain

$(n+1)n^2 > (n+1)^2$. Hence $(n+1)! > (n+1)^2$,

which completes the proof.
3. Assume for the sake of contradiction that $q > p$. By the proposition from class, there exists $r \in \mathbb{Q}$ so that $q > r > p$. By assumption, since $r > p$, we must have $q \leq r$. This contradicts the fact that $q > r$. Therefore, we must have $q \leq p$. 